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Ověření modelů pro oceňování úrokových opcí

The models verification of interest rate options valuation

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Místopřísežně prohlašuji, že jsem celou práci včetně příloh vypracovala samostatně.

V Ostravě 10. 7. 2009 Zuzana Kobzová

Poděkování

Děkuji Prof. Dr. Ing. Zdeňku Zmeškalovi za odborné konzultace a vedení diplomové práce.

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1. Introduction

The diploma thesis is focused on interest rate options. This issue falls within the financial derivatives market. The financial derivatives are quite new terms, but theirs importance is not certainly negligible. The formation of financial derivatives is dated to the 1970s, when financial market was instable; there was a high volatility of security rates, interest rates and also exchange rates. This instability brought considerable risks for all the financial market subjects. The fact resulted in efforts to find the possibility of limiting risks.

Hence, the financial derivatives occurred. They are based on the possibility to negotiate all the requirements nowadays and exercise this contract at the exact date in the future. That helps to subjects to ensure against the unfavorable development on the financial markets.

This principle has been applied previously, but since the 1970s there is a fundamental change in the structure of these transactions, the ways in trading and also new products and their combinations are formed. These new instruments also have become an attractive tool to ensure financial market risks. But this is not the only possibility how to exploit derivatives; some subjects use them for speculations and try to benefit from the development on the financial markets.

The main aim of the diploma thesis is to verify and compare the chosen models that are possible to use for interest rate options valuation. The aim is realized using the example of zero-coupon bearing bond.

The first chapter is simple introduction and also the adumbration of the diploma thesis.

The second chapter is focused on general characteristic of bonds and on interest rate options, mainly on theirs history, principles, trading and using.

The third chapter includes fundamentals about yield curves, which are necessary to know for elaboration and also describes the methods, which are used for interest rate options valuation. It is explained how to valuate the derivatives using the Black□s model, using the models of the short rate (The Vasicek model, the Cox, Ingersoll, and Ross model, the Ho-Lee model, the Hull-White model etc.) and also using numerical models (binomial and trinomial models).

The fourth chapter is devoted to construction of the yield curve and to valuation the option on the zero-coupon bearing bond using the chosen models.

The fifth chapter involves conclusion and future view on the financial derivatives market.

2. Characteristic of interest rate options

2.1 General characteristic of bonds

For the elaboration of the practical part is necessary to define the basic principles of the bonds.

2.1.1 Bonds

A bond is a debt security, in which the authorized issuer (the borrower) owes the holders (the lenders) a debt and, depending on the terms of the bond, is obliged to pay the interest (the coupon) and/or to repay the principal at a later date, termed maturity. A bond may also involve the right of the exchange for another bond or for share (an exchangeable bond), or certain prior rights (a priority bond).

Bonds are considered to be a safer investment form. Bonds are issued by public authorities, credit institutions, companies and supranational institutions in the primary markets. The most common process of issuing bonds is through underwriting. In underwriting, one or more securities firms or banks, forming a syndicate, buy an entire issue of bonds from an issuer and re-sell them to investors. The security firm takes the risk of being unable to sell on the issue to end investors. However government bonds are instead typically auctioned.

Bond markets are large, because governments often issue such securities to obtain funds as a result of deficit funding. Bonds are governed by the law of bonds, in the Commercial Code and the Securities Act.

2.1.2 The most important bond features

For the bonds are the most important features.

- *Nominal, principal or face value* - the amount on which the issuer pays interest, and which has to be repaid at the end.
- *Maturity date* - the date on which the issuer has to repay the face value. As long as all payments have been made, the issuer has no more obligations to the bond holders after the maturity date.
- *Coupon* - the interest rate that the issuer pays to the bond holders. Usually this rate is fixed throughout the life of the bond. It can also vary with a money market index, such as PRIBOR, LIBOR.

Considering purchasing bonds for the purpose of creation a portfolio, these are basic features of bonds.

- Higher yield than for bank deposits, but lower than for shares.
- Lower risk than for shares because the interest payoff is not dependent on the issuer's management, as in the case of dividends when investing in shares.
- Higher liquidity, often due to the lower risk and due to the consideration bonds as a safer form of investment.

2.1.3 Types of bonds

The law defines the specific types of bonds.

- *Mortgage bonds* - bonds which nominal value plus interest is covered by claims of mortgage loans.
- *Government bonds* - bonds issued by the Government.
- *Municipal bonds* - bonds issued by the municipalities (e.g. a state, city, local government, or their agencies).
- *Exchangeable and priority bonds* - bonds with the right to exchange the bond for another security of the issuer.
- *Subordinated bonds* - bonds that have a lower priority than other bonds of the issuer in case of liquidation. In case of bankruptcy, there is a hierarchy of creditors. First the liquidator is paid, then government taxes, etc. The first bond holders in line to be paid are those holding what is called senior bonds. After they have been paid, the subordinated bond holders are paid. As a result, the risk is higher.
- *Collective bond* - a bond that presents a complex of each issue of the bonds, which are within the emission period underwritten in subscription list. Each collective bond is a separate bond issues.

Bonds can be also divided due to the maturities. Maturity of **short-term bonds** does not exceed 1 year and of **long-term bonds** is longer than 1 year.

Other types of bonds are bonds with **fixed interest rate (fixed rate bonds), variable interest rate (variable coupon bonds)** and **zero-coupon bonds**.

The owner of **fixed rate bonds** receives interest at certain dates during whole life of the bond. Interest is determined by a percentage of face value of the bond.

Variable coupon bonds are issued for face value and their maturity is more than 1 year. The ownership of these bonds warrants regular yield between the emissions and maturity.

Regular coupons are paid in annual or biannual intervals and the amount is determined by the current market interest rate (e.g. PRIBOR) plus the predetermined margin given by the issuer. At the end of the maturity, the issuer has to pay the last coupon and also the face value of bonds.

Zero coupon bonds don't pay any interest. They are issued at a substantial discount to the face value. The bond holder receives the full principal amount at the maturity day.

2.1.4 The bond market price

Present value of the bond is calculated as the sum of the present value of all future yields of the bonds, including the face value,

$$PV = \sum_{t}^{T-1} \frac{C_t}{(1+r_t)} + \frac{C_T + FV}{(1+r_T)^T}. \tag{2.1}$$

The bond market price (P) is sometimes referred to as a dirty price. The reason is that P can be divided into a clean price, which means the bond price irrespective interest to payments of the coupon, and accrued interest, which represents interest since the last coupon payment day. (AI),

$$P = dirty\,price = clean\ price + AI\ . \tag{2.2}$$

A reason to distinguish the clean price and the dirty price is the ridged shape of the price dependency on the maturity.

The clean price is given by quoted price rate, which represents percentage of the clean price in the bond face value (FV),

$$clean\ price = quoted\ price\,rate \cdot \frac{FV}{100}. \tag{2.3}$$

Following these figures the bond market price may be determined as:

$$P = quoted\ price\,rate \cdot \frac{FV}{100} + AI\ . \tag{2.4}$$

2.2 History of interest rate options

The beginnings of derivative transactions are dated into the 12th century. Firstly, they appeared on the commodity exchanges in Italy and the Netherlands. Over time, the center of derivatives trades became London. The huge expansion of commodity derivatives occurred in the second half of the 19th century in Chicago at the Chicago Board of Trading and in New York at New York Produce Exchange and New York Coffee Exchange. Originally, the option transactions entered individually - usually by negotiation between broker who is representing the buyer and the seller of the option. It means options transactions were over the counter (OTC) transactions.

The advantage of these direct transactions is that a financial institution can claim the derivative product according to the specific needs of the client. The elements of such a product can be “nonstandard" and they can more accept financial needs of the client, instead of following rules of "standardization", by which are abided the regulations for quotation of securities on organized exchanges. On the other hand, it is obvious that the possibility of one or other party sell this non-standard contract to a third party is small. For example the writer of the option who would like to close the position before the maturity of the option, would have to find a third party who would be willing, after payment of option premium, to undertake the obligation. But it is difficult to find later a third party, because a majority of the OTC option products are claimed according specific requirements of two members of contract.

The formation of instruments that are currently indentified as financial derivatives lasted for several centuries. Primitive derivative transactions in the form of option businesses were evolved in the beginning of the 18th century on the Paris stock exchange. Due to the opinions during this period that speculation on the markets had a destabilizing effect, these transactions were prohibited. Contracts that were similar to optional one, appeared on the New York Stock Exchange at the turn of the 19th and 20th century. Future contracts were considered to be one of the reasons for excessive speculation on the stock exchanges in the 1920s and that is why they were prohibited from the period of the Great Depression till the 1970s.

The standardized financial derivatives occurred since the1970s. The first standardized financial derivatives were currency futures, traded in 1972 at the Chicago Mercantile Exchange. There was a beginning of stock option trading on the Chicago Board of Trade one year later. Since then, the market of financial derivatives grew up significantly. New optional exchanges and many different types of options contracts appeared. We can consider as reasons for this progress increasing volatility of exchange rates and interest rates. The other significant impact was the development of methods for risk management.

The value of financial derivatives has increased rapidly. The most involved in trading are banks, pension funds, insurance companies, investment and mutual funds, large corporations, exporters and importers.

The largest volume of contracts is dealt with interest rate derivatives. Currency and share derivatives have a relatively less importance. The biggest exchange of financial derivatives is Eurex, followed by Chicago exchanges (CBOT - The Chicago board of trade, CBOE - The Chicago Board Options Exchange). Branches of foreign banks and middle-sized banks are the most participated in derivatives contracts in Czech Republic.

2.3 Principles of option contracts

There are four basic derivatives: forwards or futures, swaps and options contracts. Options are fundamentally different from forward and futures contracts. An option gives a holder of the option the right to do something, but the holder does not have to exercise this right. This is the right to buy (the call option) or to sell (put option) the underlying asset. The price in the contract is known as the exercise price or strike price, the date in the contract is known as the expiration date or maturity. American options can be exercised at any time up to the expiration date. European options can be exercised only at the expiration date itself. By contrast, in a forward and futures contract, the two parties have committed themselves to some action.

Actually, it is a contract to which it is bound by law to exchange the underlying instrument at an exact date in the future; whereas the settlement is longer than practice on the spot market.

It may include exchanging a fixed amount of cash in one currency for unknown amount of cash or debt securities, deposit, credit or loan in the same currency (interest rate options or credit options), exchanging fixed amounts of cash in one currency for a fixed amount of cash in another currency (currency option), for the stock (stock options) or a commodity instrument (commodity options).

Option premium is usually payable at the moment of option conclusion (in this case, the option premium is the same as the value of option at the time conclusion). There are also options where is the possibility to pay the premium later, usually at maturity of options. This option premium is higher than the value of an option at the moment of its conclusion, because we have to calculate on accrued interests derived from risk interest rate, that occur in the period between the conclusion of an option and payment of an option premium. This interest (or reference rate, from which interest is derived) is determined at the moment of the option conclusion. In any case, a price of an option is set at the moment of the option conclusion, and it is based on market conditions at that time.

The option premium consists of two components.

- *Intrinsic value of options* - shows the convenience of immediate use of the option. This means that it shows a profit, which the owner would obtain by immediate use of option (i.e. by purchase, respectively by selling the underlying asset for the strike price) and by the current compensation trade on the stock market (sales, respectively. buying the underlying asset for the spot rate). Options have thus intrinsic value, if such a transaction can be ensured.

- *Time value of options* - it is reflecting option supply and demand. It's basically a type of commission, which occurs during the remaining time to maturity due to the changes of market conditions, so that the use of options has been profitable.

Three basic factors influenced value of premium.

- 1. factor - the difference between the option price and prompt rate at the moment of the contract conclusion. The more convenient option rate compared with spot rate is, the higher the option premium is.
- 2. factor - the length of option maturity. The longer option maturity is, the more likely movement prompt rate can be. It means that there is also more likely to realize option profits or losses.
- 3. factor - the type of option. American options can be exercised at any time up to the expiration date. European options can be exercised only at the expiration date itself. Therefore, the premium for American options is higher than for European one.

2.4 Basic terminology of options

Understanding the options required to figure out the special market terminology. According to the kind of the underlying instrument can be options divided into spot options (the holder has the right to purchase or sell the underlying instrument directly), or futures options (the holder has the right to purchase or sell the underlying instrument through the futures).

The option purchase is often called “buying volatility”. Then the buyer of an option is called “long in volatility”. On the contrary, the selling option is often called “selling volatility”). Then the seller is called “short in volatility”.

Seller of an option is also known as writer (drawer) and the act of selling is called writing an option. In the case of a call option the option holder has the right to buy the underlying instrument by a certain date for certain price. The holder can exercise this right, it is not obligatory! Each option contract includes the price for which the holder has the right to buy the asset. The price in the contract is known as the exercise price or strike price.

In addition, options are usually divided into call options put options. A long call option gives the right to holder to buy an underlying instrument for a certain price and this right lasts for certain period. Partner is the seller of a call option (a short call option). Long put option gives the right to holder to sell an underlying instrument for certain price and this right lasts for certain period. Partner is the seller of a put option (a short put option).

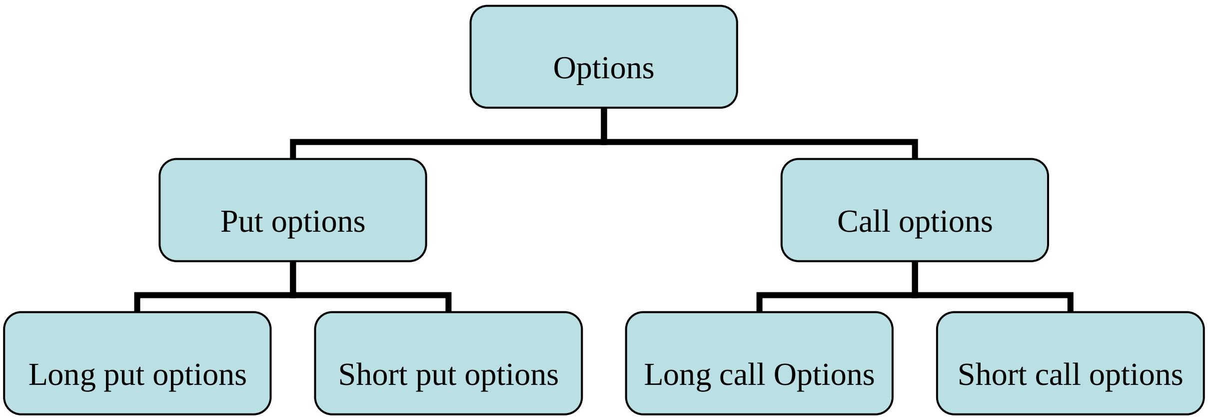

Chart 2.1 Types of options

Dividing options into call options and put options is clear only for stock and commodity options. In the case of currency options is not clear. For example, a call option CZK/USD is at the same time a selling option USD/CZK. In the case of interest rate options is the basis for this dividing dependence of yield/profit on interest rate. From a risk management point of view has this dividing no meaning (any option is a call option and a put option at the same time). For example, the option of purchasing an exact commodity for exercise price can be seen in terms of commodities as a call option and in terms of payment for the exercise price a put option.

2.5 Trade in options

Options are traded on the OTC market as well as on the derivatives exchanges. These exchanges are organized similarly to the futures exchanges. There exist a buyer and a seller for each contract in both markets. The OTC markets and exchange markets are different, on exchange markets are traded only standard options in reference to the underlying instrument, the maturity or exercise price. Result from purchase and selling of the same options on exchanges gives a zero-final position.

In contrast, options on OTC markets are generally going out of contracting parties' requirements. Purchase and selling of the same option on OTC markets does not give a zero-final position, but occurrence of two optional positions and this means to take two credit risks of both partners. Any of the partners may not meet his obligations resulted from one or other option contracts.

To buy an option, trader needs an account with brokerage firm, whose broker is a member of the exchange. To close a transaction is as easy as buying or selling shares. An option buyer pays for the option at the conclusion of a contract, so there are no further worries about cash flow resulted from such sale. For the writer is the situation of selling option more complicated. The fact that the writer sold the option, he agreed to deliver e.g. the share for certain price if the buyer decides to exercise the option. This means that the writer may need substantial financial resources to fulfill its obligation.

A trader is represented by broker on the stock exchange. Therefore, a broker wants to be sure that the writer is able to fulfill its obligation. Since the writer does not know its obligation at the time of selling options, a broker requires a certain financial guarantee.

In the case of a call option, writer can already hold the shares and such shares may pawn by the broker. Selling a call option against shares, which writer holds, is called the selling of a covered write or a covered call option. This ensures to a broker fully protection, because the shares, which may be necessary to provide, are in the holding of a broker. If the writer of call option does not hold the underlying shares, it is known as the selling of write uncovered, uncovered call or naked call option. In these cases, the broker may require some additional margin to ensure that the trader has sufficient financial resources to cover all obligations.

Clearing center makes settlement of transactions and thus helps to running of trading. It means, buyers and writers have no direct obligations towards a specific person or company, but to the clearing center. If the option is exercised, clearing center contacts a writer and a buyer, and takes control of the whole process of settlement. This settlement and the standardization of option contracts is the great benefit for exchanges and clearing center. Standardization of option contracts allow trader to focus on business strategy itself, without having to study the special features of the various option contracts.

2.6 Using of options

There are three types of traders at the markets with financial derivatives - hedgers, speculators and arbitrageurs. They use these instruments for certain purposes.

2.6.1 Hedgers

Hedging means to make closed positions. Hedgers are trading with options to reduce their risks. In the case of interest rate options, they want to protect against unfavorable trend in interest rates. They can use options to fix their position so that it will be immune to changes in the market for specific period (e.g. in this situation, when loss from one position is covered by profit from the second position).

Hedging enables to transfer risks, which belongs among the greatest contribution of derivative markets. The subject of the market will defend its position, because he sells the risk to someone else who is willing to take it over for a premium. Hedging helps to reduce the risks of expected income and that is why traders are able to plan better their future.

Decided to use hedging it is necessary to calculate with additional costs. It includes especially qualified personnel and information systems costs. We also have to count on miscellaneous charges to brokers, stock exchange, clearing center. It is always necessary to consider carefully whether hedging is profitable and ensure that acquisition expenditures do not exceed its profit.

2.6.2 Speculators

The speculation is the opposite of hedging. Speculators take over risks by creating an open position. If the rates are growing and the speculator has an open long position, it means for him to take a profit. Of course, in the case of decreasing rates, speculator receives a loss. On the other hand, the short position is taking a profit when rates are decreasing.

The fact, that speculators take over the market risks (motivated by profit) and enter the open positions, help the liquidity of derivative markets and by thus contribute to its effective functioning. Speculators are attracted to their behaving by relatively large financial leverage. And also a small change in the value of the underlying asset can cause a profit of tens or hundreds percent. This movement is, of course, possible in both directions, and the investor may lose the whole investment.

The success or failure of the speculators is largely influenced by coincidence. Nevertheless, more successful may be those who are able to better assess the market information. Using the confidential information is criminal.

2.6.3 Arbitrageurs

Arbitrageurs are seeking a profit from price differences. They use the territorial differences in prices or differences in the prices of similar contracts in different markets. They are also looking for a profit in the case when the prices on the futures market do not correspond to the price of underlying assets to prompt market.

Arbitrageurs are negotiating with derivatives at the one market and simultaneously dealing with the opposite derivatives at another market, or trading with the underlying assets and simultaneously negotiating with derivatives.

Unlike speculators, their profit is not associated with the risk of the underlying asset, but with partners' credit risk of derivatives. Opportunities for arbitrage have not usually long duration. The market law causes a price increase, where a growing focus is, and vice versa. That is why differences in prices balance very quickly. This is a positive result of the arbitration. Investors are looking for opportunities to influence the price in the markets and indirectly, due to them prices are balancing.

Final users at the derivative markets are largely speculators, hedgers are in the minority. The majority of derivatives which are considered as hedges by final users are in fact speculations. Hedging derivatives (as well as others) with a nominal value of less than 5 million U.S. $ can not be recommended for users in any case. Since the costs of negotiation are too high, it would not be effective to negotiate on derivatives up to this limit.

Final users perform defensively in the market and it is possible to say overall, they are always loosing (whether they are speculators or hedgers). For hedgers, the loss can be considered as an insurance premium against the risk, for speculators as a failure in the gambling game. Final users are attracted to the derivative trades mainly due to the large leverage effect. The initial investment represents only a small percentage of the value of the underlying asset.

2.7 Interest rate options

Interest rate option is an agreement between two parties which gives the buyer the right to buy or sell the underlying asset at a certain date in the future for a certain exercise price. Underlying asset in case of interest rate options is interest rate or other asset which price is influenced by the trend of interest rates. The writer, in contrast to buyer, has the obligation to make a contract.

Furthermore, we distinguish the following types of options.

- A currency option is an option to exchange fixed amounts of cash in one currency for a fixed amount of cash in another currency at a specific date in the future. The agreed exchange rate is the exercise exchange rate.

- A stock option is an option to exchange fixed amounts of cash for the stock instrument at a certain date in the future. The agreed price is known as the exercise price.
- A commodity option is an option to exchange fixed amounts of cash for the commodity at a specific date in the future. The agreed price is known as the exercise price.
- A credit option differs from interest-rate options, because the variable payment depends on risk interest rate of specific subject (the reference subject). Part of the credit option is an option credit spreads (a credit option, where the payment depends on the size of two credit spreads of financial assets).
- A warrant is tradable right issued separately (naked warrant) or attached to the security (usually to bonds), which gives the owner the right to buy or sell a financial asset in the same currency for a specified exercise price at the exact date in the future or during a specific exercise period. Warrants are issued for the premiums, which correspond to options.
- A swaption is an option to negotiate an interest rate swap. A swaption buyer has the right to receive fixed interest payments and pay variable interest payments.

Among the interest rate options include:

- Options on interest rate futures,
- bond options,
- cap, floor, dollar,
- interest rate swaption,

2.7.1 Options on interest rate futures (Interest rate futures options)

An interest futures option is the right (not obligation) to enter into a futures contract at a certain futures price by a certain date and the underlying asset is an interest rate futures (e.g. T-Bonds, T-Notes, Eurodollar, etc.). Specifically, a call futures option is the right to enter into a long futures contract at a certain price; a put futures option is the right to enter into a short futures contract at a certain price. Most futures options are American; that means, they can be exercised any time during the life of the contract.

2.7.2 Bond options

A bond option is the right to buy or sell a particular bond at a certain date for a certain price.

The owner has to pay an option premium for this right. The underlying asset is a bond. There are two types of bond options: An European bond option is an option to buy or sell a bond at a certain date in future for a predetermined price.

An American Bond option is an option to buy or sell a bond on or before a certain date in future for a predetermined price.

2.7.3 Caps, floors, collars

An interest rate cap is series of interest rate options, in which the buyer receives payoff at the end of each period in which the interest rate exceeds the agreed strike rate. The buyer pays for this possibility an option premium, which is fixed at the date of the contract negotiation. The interest reference rate may be equal for all options. But it is not necessary, and then this type of cap is called a variable strike cap.

As the most important cap contract parameters are considered.

- Cap nominal value is used to derive the payoff amount. There is no shift of cap nominal value between subjects of contract.
- The maturity of cap is closed by agreement of both parties. Contracts may be concluded for a period of up to 10 years and possibly longer, when most of the accounts fall to the period from one year to five years.
- The most often is used three month interest rate period and in that period the seller is obliged to pay the difference between the interest reference rate and the agreed cap rate.
- Interest reference rates are the interbank market interest rates (LIBOR, PRIBOR, etc.).

The cap buyer protects its position against rising interest rates. Each of the individual interest rate option is called caplet.

An interest rate floor is defined analogously to a cap. An interest rate floor is a series of European put options on a specified reference rate. The seller guarantees to buyer the payoff for the difference between the interest reference rate and the agreed strike rate, if the interest reference rate falls below the agreed strike rate. The buyer pays for this possibility an option premium, which is fixed at the date of the contract negotiation. The interest reference rate may be the equal for all options. But it is not necessary, and then this type of floor is called a variable strike floor.

The most important floor contract parameters are the same as for caps.

The cap buyer protects its position against decreasing interest rates. Each of the individual interest rate option is called floorlet.

An interest rate collar (sometimes called a floor-ceiling agreement) is an instrument designed to guarantee that the interest rate on the underlying floating-rate note always lies between two levels. A collar is a combination of a short position in a floor and a long position in a cap, whereas cap rate is higher than the floor rate. The collar buyer receives payoff, if the interest reference rate at the reset day is above the agreed cap rate. If the interest reference rate at the reset day falls below the agreed floor rate, the payoff must be provided by the buyer. The collar seller, short collar holder, is simultaneously in the position of the floor buyer and cap seller. Providing the payoff, when the reference rate rises above the cap rate, and receiving the payoff, when the reference rate falls below the floor rate.

The collar buyer ensures himself against rising interest rate up to the agreed level and on the other hand, he profits from contingent falling interest rate. The reason for using the collar as an instrument providing insurance (instead of the cap), are lower costs. The collar seller ensures himself against the falling interest rate. The advantage, in comparison with the floor, are lower costs (even if the collar seller has to pay the floor premium, he receives the cap premium).

In practice, there is also different combination of cap and floor, which provides another types of insurance.

- **The zero cost collar** is a combination of cap and floor, where the cap and floor rate is chosen so that the cap premium is equal to the floor premium. Thus, these two premiums are compensated. The zero cost collar buyer ensures himself against increasing interest rate, as well as in classical collar, but with the difference that he does not pay anything.
- **Participating cap**, as well as the collar, is compound of the long cap and short floor with the same maturity, interest rate periods and the interest reference rate, but with different nominal values. The floor nominal value is lower than the cap nominal value. Participating cap buyer is fully ensured against rising interest rate, and on the other hand is participated in the eventual decline of interest rates. The buyer is fully participated in the decline of interest rate between the cap and floor rates, and partly in the decline of interest rate below the floor rate. In addition to selling the floor, he receives a premium.

There are two basic differences between providing insurance through the cap and the participating cap:

- the costs when using the participating cap is lower,
- participating cap does not allow fully participation in a possible decline of interest rates.

The main advantages of using cap, floor and collar are following:

- providing insurance due to the traders´ requirements, because these instruments are non-standard,
- fully ensure the expected fluctuations of interest rates and also allowed to participate in the possible reverse trend,
- the risk of the cap or floor buyer is limited only by the amount of paid premium and that is why it is available also for less credit subjects,
- there is a relatively liquidity market for these instruments.

2.7.4 Interest rate swaptions

Interest rate swaptions are options on interest rate swaps. The buyer of a swaption has the right to enter into an interest rate swap agreement by some specified date in the future. The swaption agreement will specify whether the buyer of the swaption will be a fixed-rate receiver or a fixed-rate payer. The writer of the swaption becomes the counterparty to the swap if the buyer exercises. Generally, the use of the identification of a swaption is reserved to refer to options that involve interest rate swaps.

There are known three types of swaptions. Each type reflects a different timeframe in which the option can be exercised.

The **American interest rate swaption**, in which the owner is allowed to enter the swap on any day, that falls within a range of two dates.

The **European interest rate swaption** is an option, in which the owner is allowed to enter the swap only on the maturity date.

The **Bermudan interest rate swaption** is an option, in which the owner is allowed to enter the swap only certain dates that fall within a range of the start (roll) date and end date.

There are several points that the buyer and the seller both agree to as part of the transaction. First, both the strike rate and the premium are considered fixed. Next, the length of the option period is set, based on terms that are acceptable to both parties. Often this is in the range of two business days prior to the start date of the underlying swap for the swaption. If amortization is involved in the options, then the two parties agree to the mode of calculation involved. Last, the seller and buyer come to terms on the frequency of payments associated with the underlying swap.

A swaption does not usually involve the single investor. Instead, it is more common for investors who deal in swaption transactions to be large corporations, banks or brokerage firms, and possibly hedge funds. The use of the swaption has a fair amount of attraction, because the strategy can be used to manage the amount of interest rate risk associated with banks and other types of financial institutions.

2.8 Embedded interest rate options

There are a lot of financial instruments, which contain so-called embedded options:

- callable bonds,
- puttable bonds,
- the early redemption privileges on fixed-rate deposits,
- the prepayment privileges on a fixed-rate loan,
- mortgage bonds.

2.8.1 Callable bonds

A callable bond is a bond, which contains provisions that allow the issuing firm to buy back the bond at a predetermined price at a certain times in the future. The holder of such a bond has sold a call option to the issuer. The strike price or the call price in the option is the predetermined price that must be paid by the issuer to the holder to buy back the bond.

Part of the agreed terms is also the period of time during which the bond cannot be called. Callable bonds usually cannot be called for the first few years of their life. In case of being paid off earlier at any time during their life, the embedded options have the character of the American call options. When the possibility to be called is time limited, then the embedded options are European call options. It will be profitably for the bond issuer to exercise the option, when the market interest rate, for which it is possibility to borrow funds, falls below the rate of the bond issue.

2.8.2 Puttable bonds

Puttable bond contains provisions that allow the holder to demand early redemption at a predetermined price at certain times in the future. The holder of such a bond has purchased a put option on the bond as well as bond itself. Because the put option increases the value of the bond to the holder, bonds with put features provide lower yields than the bonds with no put features.

2.8.3 The early redemption privileges on fixed-rate deposits

The early redemption privileges on fixed-rate deposits are a bank product with embedded option. The financial institution, which gives to savers early redemption privileges, is also the put option issuer (it is the same as in case of puttable bonds).

In this case, the option buyer does not pay in advance the premium; however the bank requires a penalty for early withdrawal, which is paid only at the exercise day.

The amount of the penalty must be taken into account when calculating the value of an option. When the bank does not require a penalty for early withdrawal, such a deposit will be less profitable due to the put options, which increases the value of the deposit for the depositors.

2.8.4 The prepayment privileges on a fixed-rate loan

The prepayment privileges on a fixed-rate loan are also a bank product with embedded option. The right of the debtor to pay a loan in advance presents a call option (it is based on a similar principle as an option in the case callable bonds). The value of this option influences the decision on the choice of the most profitable forms of funding, when comparing loan products of various banks. An important indicator is net present value of loans. This value is determined by discounting the cash flows associated with the loan over its duration. When comparing the individual products, it is selected the one with the lowest net present value. In case that the bank offers the possibility of prepayment under certain conditions, this fact should be included in economic considerations. The right of prepayment presents a call option, which the financial institution issues and sells to the debtor together with the loan. It will be profitable for debtor to exercise the option, when the market interest rate, for which it is possibility to borrow funds, falls below the fixed rate loan.

2.8.5 Mortgage bonds

A bond secured by a mortgage on property. Mortgage bonds are backed by real estate or physical equipment that can be liquidated. Mortgage bonds are usually considered high-grade, safe investments and safer than unsecured bonds. If an issuer in default has both secured and unsecured bonds outstanding, secured bondholders are paid off first, then unsecured bondholders. Because unsecured bonds carry greater risk than secured bonds, they usually pay higher yields.

Financial derivatives are varied financial instruments, which are constantly evolving according to the needs of investors who use them. The introduced characteristics and classifications are considered as fundamental, and may further diversify into various combinations among instruments themselves, or among the underlying assets.

3. Valuation models of interest rate options and the models of the yield curves

3.1 Yield curves

Fixed income securities are influenced mainly by the trend of yield curves. The yield curve represents the dependence of yields on the time to maturity. The shape of the yield curve corresponds to the future interest rate course. The yield to maturity is defined as the internal rate of return from the cash flow and the market price of fixed income securities (P),

$$\sum_t CF_t \cdot (1+r_t)^{-t} = P. \tag{3.1}$$

The yield curve can be constructed in many different ways. We distinguish spot and forward yield curve. The yield can be generally depicted as ${}_a y_{bc}$, where a denotes the moment of decision-making, b means the beginning and c the end of the interval from which the yield is calculated.

The spot yield r is a yield, which is determined within the interval, which starts at the moment of decision-making,

$${}_0 y_{0,t} = r_t. \tag{3.2}$$

The forward yield is a yield, which is always determined for an interval in the future,

$${}_0 y_{t_1,t} = f_{t_1,t}. \tag{3.3}$$

Sometimes, we consider a short-term yield s, which means the forward for one period,

$${}_0 y_{t_1,t} = f_{t_1,t} = s_t, \tag{3.4}$$

where $t - t_1 = 1$.

3.1.1 Correlation between spot and forward yield curves

Correlation between spot and forward yield curves is illustrated in chart 3.1.

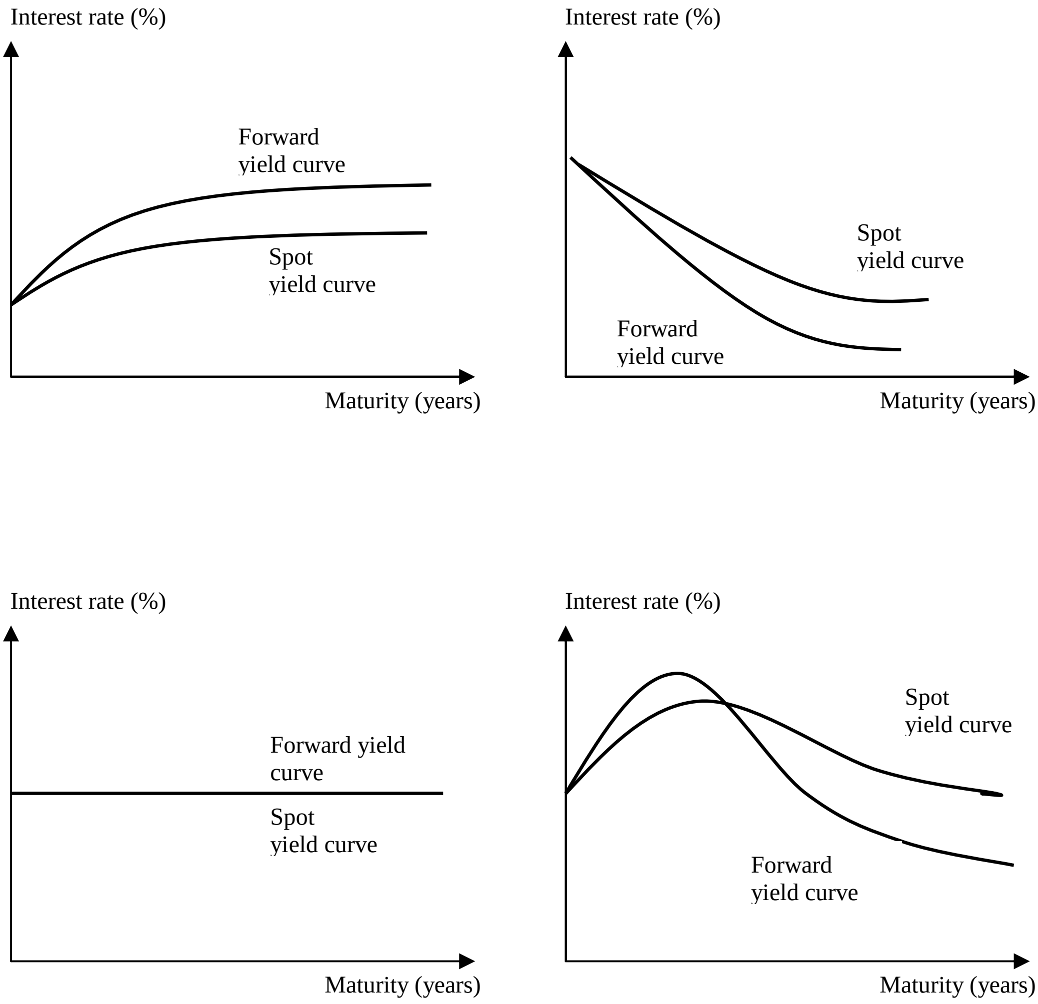

Chart 3.1 Correlation between spot and forward yield curves

It is apparent from the picture that in the case of upward sloping spot yield curve is the forward interest rate for each maturity higher than the spot interest rate. If the spot yield curve is downward sloping, the forward interest rate for each maturity is lower than the yield to maturity.

3.1.2 Construction of yield curves

The following procedure of construction of yield curves was suggested by Andrea Resti, Andrea Resti (1999).

The first step is to determine the transition matrices for each year. The basic transition matrix is necessary to extended by the likelihood of default and the likelihood of default migrating to any other rating. In this case, it can be concluded that the likelihood of default migration is zero according to the impossibility of migrating to any other rating. The opposite case is the company's likelihood of default, when the company is already been in insolvency. Then the value of the likelihood of default is 1 (100%). The result is the matrix T :

$$T = \begin{vmatrix} T_V & t_d \\ 0 & 1 \end{vmatrix}. \tag{3.5}$$

From the one-year transition matrix can be inferred two-year transition matrix. This derivation is determined by multiplying matrixes T and T :

$$T^2 = T \cdot T = \begin{vmatrix} T_V^2 & (1+T_V)t_d \\ 0 & 1 \end{vmatrix}, \tag{3.6}$$

and then also n-year transition matrix,

$$T^n = \begin{vmatrix} T_V & \sum_{i=0}^{n-1} T_V^i t_d \\ 0 & 1 \end{vmatrix}, \tag{3.7}$$

where T^n is the company's likelihood of default at the risk horizon n-years for each rating category.

First step for determination the yields to maturities is to recalculate the each transition matrix according the figures 3.6 – 3.8 and to define the risk free rates for each year. For the risk free rate was chosen the 1Y PRIBOR on the date 31.3.2009. The value of this rate was overtaken from the webpage of the Czech National Bank[1].

Based on the determined risk free rates and known likelihood of default p_n^i is necessary to calculate the spot interest rate r_i for the company with rating i. The important assumption is that the investor requires the option premium for running the risk. The one-year spot interest rate r_i is calculated thus:

$$(1+r_1^i)(1-p_1^i) + p_1^i RR = 1 + r_i^F, \tag{3.8}$$

where RR is a recovery rate in case of debtor's default and its value 51,13 % was determined empirically from the face value and r_i^F is one-year risk free rate.

Subsequently, we have to define two-year interest rate, which is possible to infer from the following formula:

[1] www.cnb.cz

$$p_1^i RR \frac{(1+r_2^F)^2}{(1+r_1^F)} + (p_2^i - p_1^i) RR + (1+r_2^i)^2 (1-p_2^i) = (1+r_2^F)^2, \tag{3.9}$$

and after modification we get the final formula for two-year spot interest rate:

$$r_2^i = \sqrt{\frac{(1+r_2^F)^2 - p_1^i RR \frac{(1+r_2^F)^2}{(1+r_1^F)} - (p_2^i - p_1^i) RR}{1-p_2^i}} - 1. \tag{3.10}$$

If we generalize the equation above, we can determine the spot interest rate for *n*-years:

$$r_n^i = \sqrt{\frac{(1+r_n^F)^n - RR \sum_{j=1}^{n} \left\{ p_{j-1}^i \frac{(1+r_n^F)^n}{(1+r_{j-1}^F)^{j-1}} + (p_j^i - p_{j-1}^i) \right\}}{1-p_n^i}} - 1. \tag{3.11}$$

Known the spot interest rates, we are able to calculate the forward interest rates according to the formula:

$$f_t = \frac{(1+r_t)^t}{(1+r_{t-1})^{t-1}} - 1. \tag{3.12}$$

3.1.3 Theories of yield curves

There are three theories, which generally explain the shapes of yield curves:

- the expectation theory,
- the liquidity preference theory,
- the market segmentation theory.

According to **the expectation theory**, long-term interest rates reflect expected future short-term interest rates. Present forward interest rate is the best estimation of future spot interest rates for the corresponding forward rate. The theory assumes that market traders do not prefer any specific maturity date; the investors want to achieve the highest yield.

The liquidity preference theory is based on the argument that investors prefer short-term securities before long. The reason is the risk aversion. Short-term bonds have less interest rate risk, and therefore investors require higher yield when they invest in long-term securities. Prices of long-term securities are more sensitive to movements in interest rates. According to this theory, the forward interest rate exceeds the expected future spot interest rates.

According to **the market segmentation theory**, there is no relation between the short, mid-term and long term interest rates. The market for bonds is dominated by large financial investors with strong preferences, relating to the maturity of the bonds. The market is divided into several segments according to maturity, they require. In these segments, yields are determined by supply and demand. Short-term interest rate is determined by supply and demand in the market of short-term financial instruments, the mid-term interest rate is determined by supply and demand in the market of mid-term financial instruments, etc.

None of these theories can fully explain the real yield curve. The general fact remains that the forward interest rates give important information about the future course of interest rates.

3.2 Valuation models of interest rate options

Valuation of interest rate derivatives is more complicated compared to other types of derivatives, in particular for the following reasons.

- Probability behavior of individual interest rates in time has its own specifications, and that is why it is more difficult to describe it by the probability functions in comparison with the share price or currency rates.
- For valuation interest rate derivatives is often a necessary to create a model that describes the probability behavior of future interest rates on the whole yield curve.
- Volatility of individual points along the yield curve is different.
- Interest rates are used for determination of the pay off function of the interest rate derivatives, and also for discounting while determine the present value of the derivative.

There are two approaches for the valuation of derivatives on interest rates. The first approach is based on the assumption that the evolution of the underlying asset has the lognormal distribution. **Black's model** is based on this assumption. This model is suitable for derivatives, which payoff depends on the value of the one variable (e.g. interest rate or bond price).

On the other hand, the second approach rejects the assumption of lognormal distribution of the underlying asset and respects the fact that the development of underlying asset is influenced by the evolution of interest rates in the economy. In this case, it is used **models of the short rates.**

3.2.1 Black's model

Black's model was designed as the reaction to very popular tool - Black-Scholes model, which enables only valuation of stock options and due to needs to extend this model for valuation of other derivatives. Black's model was originally developed for valuing options on commodity futures. Later, it has been extended so it can be used to value options on foreign exchange, options on indices and options of futures contracts. Traders have found flexible ways of using the model to reflect their needs so that it covers also valuing of interest rate derivatives.

Generally, this model is suitable for derivatives, which payoff depends on the value of the one variable (e.g. interest rate or bond price). Exactly, it is suitable to use them for valuing of caps, European bonds, European swaptions etc. The disadvantage is that model does not respect evolution of interest rates in time. That is why it is not possible to use it for valuing American options, financial instruments with embedded options etc.

Black's model is based on following assumptions:

- interest rates are not stochastic,
- probability distribution of value of the underlying asset (V_T) is log-normal,
- volatility of logarithm of value of the underlying asset $(ln V_T)$ is equal to $\sigma \cdot \sqrt{T}$,
- there are no taxes and transaction costs,
- perfect capital market,
- pricing in continuous time,
- the interest rate is constant,
- the option price does not depend on expected returns,
- forward price of underlying asset is equal to futures price of underlying asset, which is defined as expected value of V_T with risk-free rate.

Consider the European call option; the value can be defined as follows:

$$c = e^{-r \cdot T} \cdot [F_T \cdot N(d_1) - X \cdot N(d_2)], \tag{3.13}$$

where $$d_1 = \frac{ln\left(\frac{F_T}{X}\right) + \sigma^2 \cdot \frac{T}{2}}{\sigma \cdot \sqrt{T}} \tag{3.14}$$

and $$d_2 = \frac{ln\left(\frac{F_T}{X}\right) - \sigma^2 \cdot \frac{T}{2}}{\sigma \cdot \sqrt{T}} = d_1 - \sigma \cdot \sqrt{T}\ . \tag{3.15}$$

c is the value of the option, F futures price of V for a contract maturing at time T, T time to maturity of the option, V_T value of V at time T, F_T value of F at time T, X strike price of the option, r interest rate for maturity T and σ volatility of F.

The value, p, of the corresponding put option is given by:

$$p = e^{-r \cdot T}\left[X \cdot N(-d_2) - F_T \cdot N(-d_1)\right]. \tag{3.16}$$

The underlying asset (V) is mostly an interest rate, bond price or spread between interest rates.

The Black's model to price the European option on coupon bearing bond

A European bond option is an option to buy or sell a bond for a certain price (X), at a certain time (T). A common assumption in valuation bond options is that the bond price is lognormal at time T. Then figures (2.1) and (2.4) can be used. F_T is the forward bond price and the variable σ is the volatility of F, so that $\sigma \cdot \sqrt{T}$ is the standard deviation of the logarithm of the bond price at time T. Forward price F_T can be calculated from today's spot bond price (B_0), using the formula:

$$F_T = (B_0 - I_0) \cdot e^{r \cdot T}, \tag{3.17}$$

where B_0 is today's spot bond price, F_T forward price for maturity T, I_0 present value of the coupons that will be paid during the life of the option and r the interest rate for maturity T.

The variables F_T, B_0, X are characterized as dirty prices (it means clean price with accrued interest).

The variable σ is defined as:

$$\sigma = \frac{\sigma_{ln V_T}}{\sqrt{T}}, \tag{3.18}$$

where $\sigma_{ln V_T}$ is volatility of logarithm of bond price at maturity T, T is maturity of the option.

The chart 3.2 shows, how volatility of logarithm of bond price is changing in time. Today, in time 0, there is no doubt about bond price so that the volatility is equal to 0. At the maturity day, the volatility is also 0, if we know, that the price will be equal to the face value. During the life of the bond, the volatility has progressive and then degressive trend.

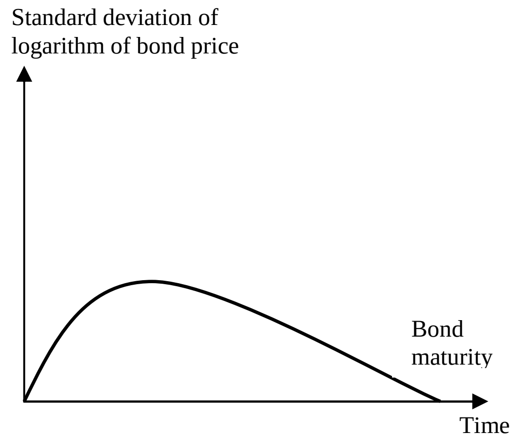

Chart 3.2 Standard deviation of logarithm of bond price at future times

3.2.2 Models of the short rate

The disadvantage of the Black's model is that it does not respect evolution of interest rates in time. That is why it is not possible to use it for valuing American options, financial instruments with embedded options etc. For the valuing of these instruments are used methods, that are based on the models of the short rate. In these models, the evolution of interest rates in time is considered as random and risk parameter. We can distinguish these models according to the chart 3.3.

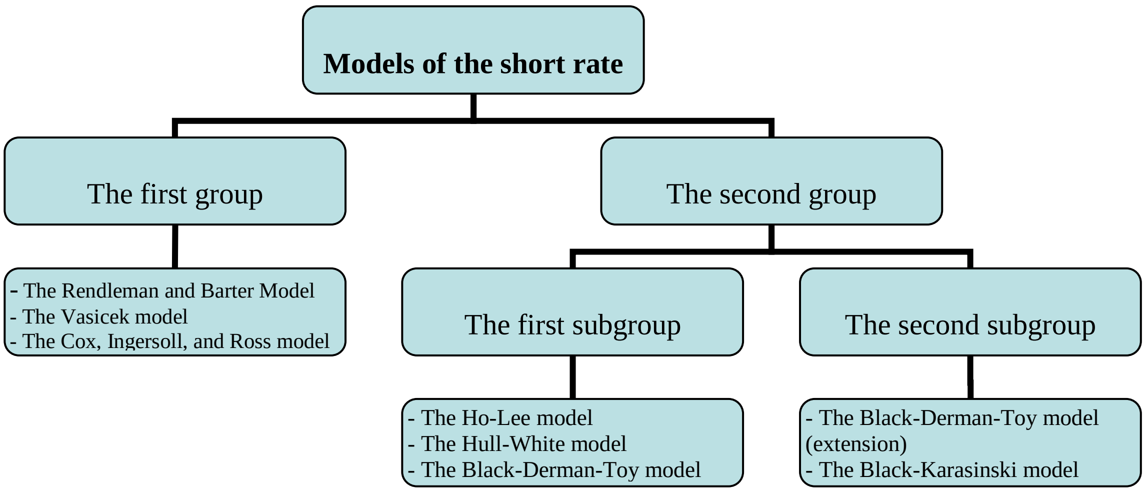

Chart 3.3 Models of the short rate

The first group takes into account the current yield curve in time of the model construction. The initial term structure of interest rates is in principle the output of the model. Based on specific parameters, we can get the approximate adaptation to the yield curves, which are practically met. This adaptation can lead to errors during the valuation.

This insufficiency is solved by the models of the second group. These models are constructed to be in correspondence with the current yield curve in time of the construction. The models are possible to distinguish furthermore due to the amount of variables in time. The first subgroup involves only one variable and the second several variables.

3.2.2.1. The general characteristic of models of the short rate

The background of the models of the short rate is based on the random evolution of the short interest rate (r) in time. We consider risk-neutral approach as relevant. The short interest rate r, at time t, is the rate that applies to an infinitesimally short period of time at time t. It is sometimes referred to as the instantaneous short rate. The random evolution $r(t)$ is described by an Ito□ process of the form:

$$dr = m(r,t)dt + \sigma(r,t)d\tilde{z} , \tag{3.19}$$

where dr is interest rate shift in time, m the instantaneous drift, σ instantaneous standard deviation, dt the length of time period, dz is specific Wiener process.

The random factor dz represents specific Wiener process:

$$dz = z \cdot \sqrt{dt} , \tag{3.20}$$

where z is the random factor from normal distribution $N(0,1)$.

3.2.2.2. The Rendleman and Barter Model

In Rendleman and Barter's model, the risk neutral process for r is:

$$dr = m \cdot r \cdot dt + \sigma \cdot r \cdot d\tilde{z} , \tag{3.21}$$

where m is mean value of interest rate yields, r interest rate, σ is standard deviation.

This model is similar to standard model of geometric Brownian motion. The process for r is of the same type as that assumed for a stock price.

The assumption that the short-term interest rate behaves like a stock is not ideal. One important difference between interest rates and stock prices is that the interest rates appear to be pulled back to some long-run average level over time. This feature is known as mean-reversion. When r is high, mean reversion tends to have a negative drift. When r is low, mean reversion tends to have a positive drift. The Rendleman and Batter's model does not incorporate mean reversion.

3.2.2.3. The Vasicek model

In Vasicek's model, the risk neutral process for r is:

$$dr = a(b - r)dt + \sigma \cdot d\tilde{z} , \tag{3.22}$$

where a,b and σ are constants. This model incorporate mean reversion and it respects an empirical fact, that interest rates regularly revert to the long-run mean b, with the velocity a. The disadvantage is that allows the interest rates to be negative (it is changed in CIR model). This fact is unrealistic.

The high interest rates (above b) has negative influence (parameter b will be negative). This leads to decreasing of the interest rates in time until the long-term equilibrium. On the contrary, the low interest rates lead to rising of interest rates in time until the long-term equilibrium. The shape of the yield curves in case of Vasicek's model can be upward-sloping, downward sloping, or slightly humped. It is shown on chart 3.4.

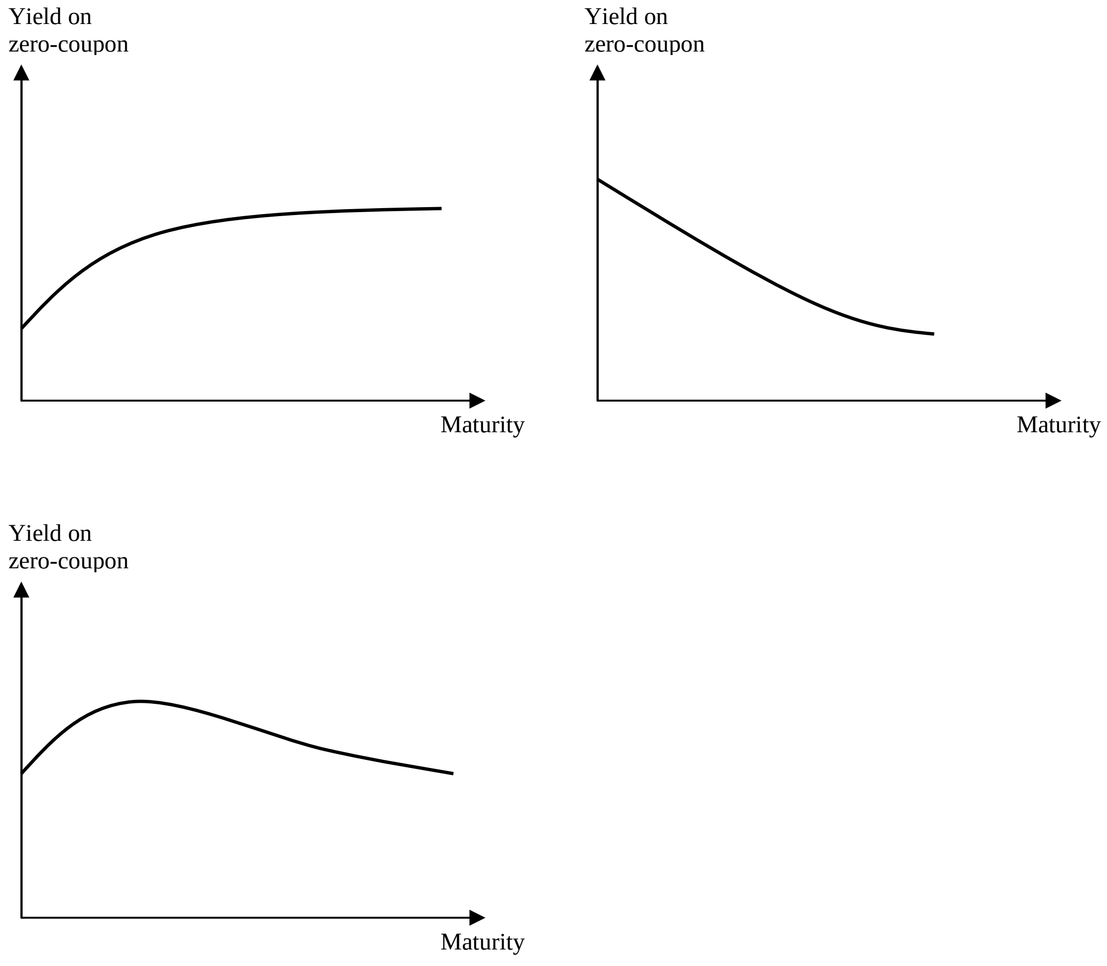

Chart 3.4 Possible shapes of term structure when Vasicek´s model is used

The Vasicek model to price the European option on zero-coupon bond

The Vasicek model can be used to obtain the following expression for the price at time t of a zero-coupon bond that pays 1 m.u. at time T:

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}, \tag{3.23}$$

where $r(t)$ is the value of r at time t,

$$B(t,T) = \frac{1 - e^{-a(T-t)}}{a}, \tag{3.24}$$

and

$$A(t,T) = exp\left[\frac{(B(t,T) - T + t)(a^2 b - \frac{\sigma^2}{2})}{a^2} - \frac{\sigma^2 B(t,T)^2}{4a}\right]. \tag{3.25}$$

The price of the European call option at time t and exercise at time T is then:

$$c = FV \cdot P(t,s)N(h) - X \cdot P(t,T)N(h - \sigma_p), \tag{3.26}$$

where

$$h = \frac{1}{\sigma_p} ln \frac{FV \cdot P(t,s)}{X \cdot P(t,T)} + \frac{\sigma_p}{2}, \tag{3.27}$$

$$\sigma_p = \frac{\sigma}{a}\left[1 - e^{-\alpha(s-T)}\right]\sqrt{\frac{1 - e^{-2\alpha(T-t)}}{2a}}, \tag{3.28}$$

and where FV is the face value of the bond, s the maturity of the bond, T the maturity day of the option, t the valuation day, X strike price of the option.

The price of European put option is calculated as:

$$p = X \cdot P(t,T)N(-h + \sigma_p) - FV \cdot P(t,s)N(-h). \tag{3.29}$$

If $a = 0$ and $\sigma_p = \sigma(s - T)\sqrt{T - t}$. (3.30)

The Vasicek model to price the European option on coupon bearing bond

The price of the coupon-bearing bond is possible to calculate from the price of the zero-coupon bond applying the Jamshidian's procedure. The assumptions are the European call option on the coupon-bearing bond with strike price X and maturity at time T and that the bond generates cash flow after the option maturity.

We define that CF_i is cash flow generates at time s_i, where $(1 \le i \le n; s_i > T)$; r^* is the value of the short rate at time T, while the value of coupon-bearing obligation is equal to strike price; X_i is the value zero-coupon bond at time T, with face value 1 m.u. at time s_i, when $r = r^*$; $P(T,s_i)$ is the zero-coupon bond price at time T with maturity at time s_i.

$$X_i = CF_i \cdot P(T,s_i), \tag{3.31}$$

$$c = FV \cdot P(0,s)N(h) - X \cdot P(0,T) \cdot N(h - \sigma_p). \tag{3.32}$$

The option on the coupon-bearing bond can be determined as the sum of the option on the zero-coupon bonds. The price of the put option is possible to calculate analogously.

3.2.2.4. The Cox, Ingersoll, and Ross model

The Cox, Ingersoll, and Ross model relieves the shortage of Vasicek´s model, where the short-term interest rate r can become negative. The risk-neutral process for r in their model is:

$$dr = a(b - r)dt + \sigma\sqrt{r} \cdot dz\ . \tag{3.33}$$

The standard deviation of the change in the short rate in a short period of time is proportional to $\sqrt{r}$. This means, as the short-term interest rate increases, its standard deviation increases.

The Cox, Ingersoll, and Ross model admit the same shapes of the yield curves as in Vasicek´s model and also the bond prices have the same general form:

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}. \tag{3.34}$$

But the functions $B(t,T)$ and $A(t,T)$ are different:

$$B(t,T) = \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + a)(e^{\gamma(T-t)} - 1) + 2\gamma}, \tag{3.35}$$

$$A(t,T) = \left[\frac{2\gamma e^{(a+\gamma)\frac{(T-t)}{2}}}{(\gamma + a)(e^{\gamma(T-t)} - 1) + 2\gamma} \right], \tag{3.36}$$

with $\gamma = \sqrt{a^2 + 2\sigma^2}$. (3.37)

The long rate $R(t,T)$ is linearly dependent on $r(t)$. This means that the value of $r(t)$ determines the level of the term structure (the yield curve) at time t. The general shape of the yield curve at time t is independent of $r(t)$, but does depend on t.

The Cox, Ingersoll, and Ross model gives the instruction how to evaluate European call and put options on zero-coupon bonds. The coupon-bearing bonds can be evaluated due to applying the Jamshidian's procedure similarly as in Vasicek´s model.

3.2.2.5. The Ho-Lee model

Ho and Lee proposed the first no-arbitrage model of the term structure. It involves two parameters: the standard deviation of short rate and the market price of risk of the short rate. The continuous-time limit of the model is:

$$dr = \theta(t)dt + \sigma dz\,, \qquad (3.38)$$

where σ is the instantaneous standard deviation of the short rate (it is constant), $\theta(t)$ is a function of time chosen to ensure that the model fits the initial term structure. The variable $\theta(t)$ defines the average direction that r moves at time t. This is independent of the level of r. It can be calculated as:

$$\theta(t) = F_t(0,t) + \sigma^2 t\,, \qquad (3.39)$$

where $F_T(0,t)$ is instantaneous forward rate for a period $(0,t)$, that is possible to determine from the current yield curve.

We suppose that $F_T(0,t)$ equals to $\theta(t)$. This means that the average direction that the short rate will be moving in the future is approximately equal to the slope of the instantaneous forward curve.

The disadvantage of the Ho-Lee model are that it does not take into the account the mean-reversion process and each of the spot and forward yield curves has the same standard deviation.

The Ho-Lee model to price the European option on zero-coupon bond

We can evaluate the European option on zero-coupon bond analogously. The price of discounted bond at time t can be defined as:

$$P(t,T) = A(t,T)e^{-r(t)(T-t)}\,, \qquad (3.40)$$

$$\ln A(t,T) = \ln\frac{P(0,T)}{P(0,t)} - (T-t)\frac{\partial \ln P(0,t)}{\partial t} - \frac{1}{2}\sigma^2 t(T-t)^2\,. \qquad (3.41)$$

In these equations, time zero is today. Times t and T are general times in the future with $T \geq t$.

The equations define the price of a zero-coupon bond at a future time t in terms of the short rate time t and the bond prices today. The partial derivation $\frac{\partial \ln P(0,t)}{\partial t}$ can be approximated:

$$\frac{\ln P(0,t+\varepsilon) - \ln P(0,t-\varepsilon)}{2\varepsilon}\,, \qquad (3.42)$$

where ε is the short time period (e.g. 0,01 of the year).

The volatility of the discounted bond at time t with maturity at T is:

$$v(t,T,\Omega_t) = \sigma(T-t)\,. \qquad (3.43)$$

The price of the European call option at time 0, with maturity at time T, on zero-coupon bond at time s is defined:

$$c = FV \cdot P(0,s)N(h) - X \cdot P(0,T)N(h - \sigma_P), \tag{3.44}$$

where $h = \frac{1}{\sigma_P} ln \frac{FV \cdot P(0,s)}{P(0,T)X} + \frac{\sigma_P}{2}$, (3.45)

$$\sigma_P = \sigma(s - T)\sqrt{T}. \tag{3.46}$$

Then the price of the put option is:

$$p = X \cdot P(0,T)N(-h + \sigma_P) - FV \cdot P(0,s)N(-h). \tag{3.47}$$

The European coupon-bearing bonds can be evaluated due to applying the Jamshidian's procedure.

3.2.2.6. The Hull-White model

Hull and White proposed the extensions of Vasicek's model that provide an exact fit to the initial term structure. Then:

$$dr = (\theta(t) - ar)dt + \sigma dz, \tag{3.48}$$

or $dr = a\left[\frac{\theta(t)}{a} - r\right]dt + \sigma dz$. (3.49)

The parameters a and σ are constants. The Hull-White model can be characterized as the Ho-Lee model with mean reversion at rate a. It means that the Ho-Lee model is then special case of the Hull-White model with $a = 0$. Alternatively, it can be also characterized as the Vasicek model with independent reversion level. At time t, the short rate reverts to $\frac{\theta(t)}{a}$ at rate a. The function $\theta(t)$ can be calculated from the initial yield curve:

$$\theta(t) = F_t(0,t) + aF(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at}). \tag{3.50}$$

The last term in this equation is usually very small, so that we can ignore it. If we does it, the drift of the process for r at time t is $F_t(0,t) + a[F(0,t) - r]$. This shows that short rate r follows the slope of the initial forward curve. In case of deviation from that curve, it tends to revert back to the curve at rate a.

The Hull-White model to price the European option on zero-coupon bond

The bond price is defined as:

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}, \tag{3.51}$$

where

$$B(t,T) = \frac{1-e^{-a(T-t)}}{a}, \tag{3.52}$$

$$\ln A(t,T) = \ln\frac{P(0,T)}{P(0,t)} - B(t,T)\frac{\partial \ln P(0,t)}{\partial t} - \frac{1}{4a^3}\sigma^2(e^{-aT}-e^{-at})^2(e^{2at}-1). \tag{3.53}$$

The equations above illustrate the price of a zero-coupon bond at a future time t in terms of the short rate at time t and the prices of bonds today.

We can also approximate the partial derivation $\frac{\partial \ln P(0,t)}{\partial t}$:

$$\frac{\ln P(0,t+\varepsilon) - \ln P(0,t-\varepsilon)}{2\varepsilon}, \tag{3.54}$$

where ε is the short time period (e.g. 0,01 of the year).

The price of the European call option at time t, with maturity at time T, on zero-coupon bond at time s is defined:

$$c = FV \cdot P(0,s)N(h) - X \cdot P(0,T)N(h-\sigma_P), \tag{3.55}$$

where

$$h = \frac{1}{\sigma_P}\ln\frac{FV \cdot P(0,s)}{P(0,T)X} + \frac{\sigma_P}{2}, \tag{3.56}$$

$$\sigma_P = \frac{\sigma}{a}\left[1-e^{-a(s-T)}\right]\sqrt{\frac{1-e^{-2aT}}{2a}}. \tag{3.57}$$

Then the price of the put option is:

$$p = X \cdot P(0,T)N(-h+\sigma_p) - FV(0,s)N(-h). \tag{3.58}$$

The European coupon-bearing bonds can be evaluated due to applying the Jamshidian's procedure.

This model similarly as Vasicek´s model involves two parameters of the volatility. The parameter σ means the standard deviation of the short rates and the parameter a the relatively deviation of the long-term and short-term rates, so the trend of the interest rates that reverts back to the long-run average level. This model provides a richer pattern of term structure movements and a richer pattern of volatilities.

The volatility at time t of the bond with maturity at time T is:

$$v(t,T,\Omega_t) = \frac{\sigma}{a}\left[1-e^{-a(T-t)}\right]. \tag{3.59}$$

The standard deviation at time t is then:

$$\frac{\sigma}{a(T-t)}\left[1-e^{-a(T-t)}\right]. \tag{3.60}$$

The Hull-White model includes the advantages of Vasicek´s model and also of the Hull-White model. This means that it involves the reversion of the interest rates to the long-run average level and also respects current term structure.

3.2.2.7. The Black-Derman-Toy model

The Black-Derman-Toy model forms the random evolution of the interest rate as:

$$d\ln r = \theta(t)dt + \sigma d\tilde{z}\ . \tag{3.61}$$

This model is practically identified as the Ho-Lee model; the only difference is that the initial variable r is substituted by logarithm. This provides that the generated short rate is always positive. The model with using Ito□´s lemma gives this equation:

$$dr = \left[\theta(t) + \frac{1}{2}\sigma^2\right] r \cdot dt + \sigma \cdot r \cdot d\tilde{z}\ . \tag{3.62}$$

3.2.2.8. The Black-Karasinski model

The Black-Karasinski model allows only positive interest rate and the evolution is illustrated:

$$d\ln r = (\theta - a\ln r)dt + \sigma \cdot d\tilde{z}\ . \tag{3.63}$$

The variable $\ln r$ follows the same process as r in the Hull-White model, but the difference is that it is lognormal (in the Ho-Lee and Hull-White models is normal). Also, the Black-Karasinski model does not have so much analytic tractability as Ho-Lee or Hull-White.

3.2.3 Numerical models for valuation of interest rate options

Numerical models are used for valuing the interest rate derivatives only approximately. These models involve methods, which are based on the construction of decision trees (binomial and trinomial tree). The advantage of these models is possibility of valuing every interest rate derivative. On the other hand these models are difficult to figure out and also time-consuming due to the big amount of calculation.

A decision tree represents random evolution of underlying asset in discrete time. The first step for valuing interest rate derivatives is to determine random walk of short rate r using the binomial or trinomial model. We usually assume, that short rate is evolved due to the same stochastic process as in the case of generation of random evolution r in continuous time. Then, we have to define the value of underlying asset in each node of a model. This step is followed by calculating of intrinsic value and price of derivative in each node and also the strike price.

Charts 3.5 and 3.6 show models of decision trees.

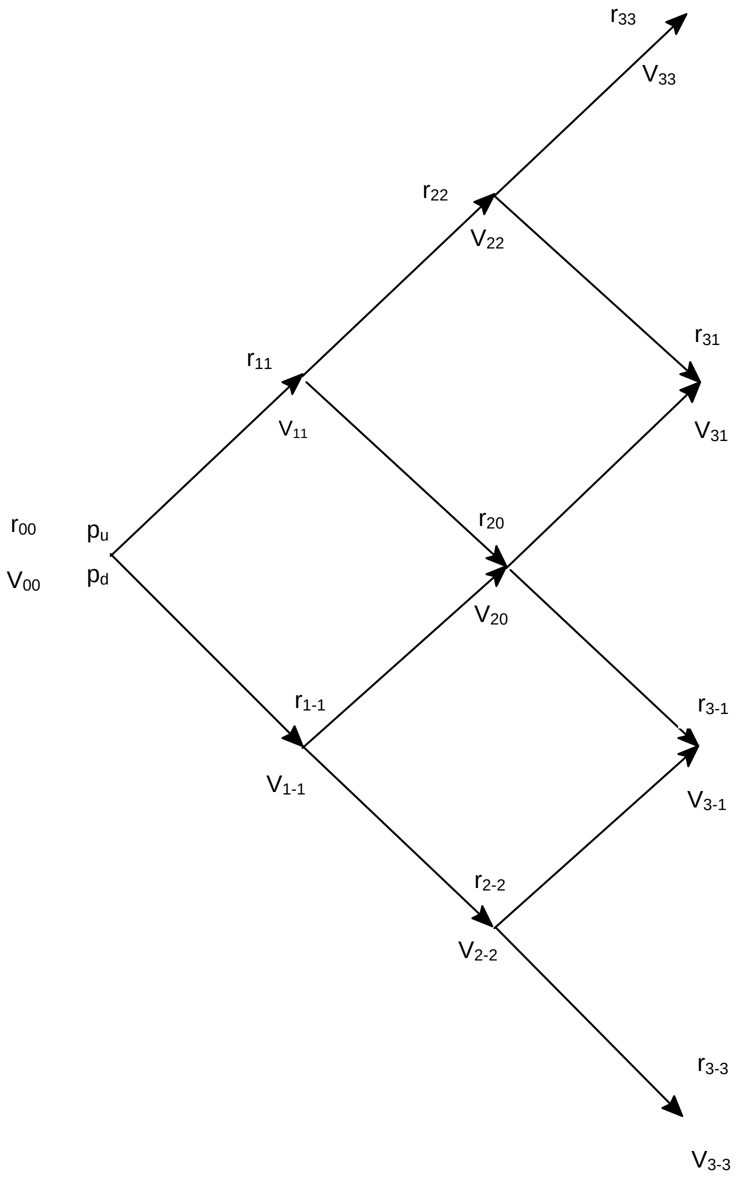

Chart 3.5 Binomial model

(t,s) is a definition of node, where t is time and s is state at certain time, Δt the length of one step, $r_{t,s}$ short rate for one period (Δt in time t and in state s), $V_{t,s}$ value of underlying asset in node (t,s) and p_u, p_d is risk-neutral probability between nodes (moving up, moving down).

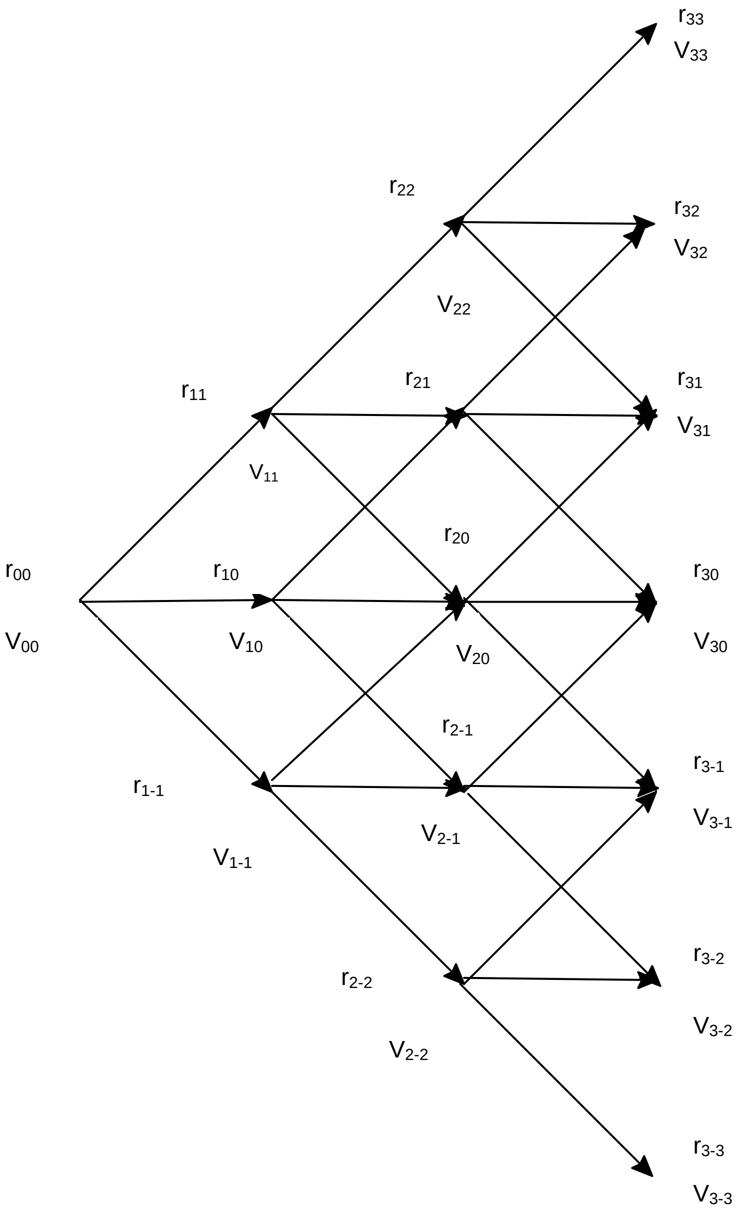

Chart 3.6 Trinomial model

The assumption is risk-neutral valuation, when the value of underlying asset in time t and position j expressed as:

$$V_{t,s} = \frac{1}{1+r_{t,s}}\left(p_u \cdot V_{t+1} + p_d \cdot V_{t+1,s-1}\right) + CF_{t,s}, \tag{3.64}$$

in case of binomial model and compound interests,

$$V_{t,s} = \frac{1}{1+r_{t,s}}\left(p_u \cdot V_{t+1} + p_m \cdot V_{t+1,s} + p_d \cdot V_{t+1,s-1}\right) + CF_{t,s}, \tag{3.65}$$

in case of trinomial model and compound interest.

The condition needed for using these models is that the arbitrage opportunities do not exist. When we want to define exact short rate in each node, we have to come out of discounted value of bonds in time 0 and maturity in time n. The one way how to determine this value is to use backward recurrent procedure starting at maturity. The bond price can be defined with respect to particular nodes as:

$$P_{t,s}(n) = \frac{1}{1+r_{t,s}}\left[p_u \cdot P_{t+1,s+1}(n) + p_d \cdot P_{t+1,s-1}(n)\right], \tag{3.66}$$

In absence of arbitrage it must hold that:

if $P_{t,s} \geq 0$, than $\left(P_{t+1,s+1} \geq 0 \wedge P_{t+1,s} \geq 0 \wedge P_{t+1,s-1} \geq 0\right)$. Since each bond pays a positive face value at maturity, each term of formulation is positive. Hence, the condition in absence of arbitrage must always hold.

3.2.3.1. Binomial model

Suppose, that short-term interest follow the discrete version of Ho-Lee model, and then the binomial model for determination of short term rate can be defined as:

$$r_{t,s} = a_t + b_t \cdot s\,, \tag{3.67}$$

where a_t is the average rate in time t and b_t is the volatility.

The volatility is possible to calculate from $b_t = 0{,}5\sigma$. This parameter is constant in the case of Ho-Lee model. The parameter a_t helps to realize the equality of spot and future short-term interest rates, so that the model can be in accordance with input yield curve. It means that a_t is variable in time. Risk-neutral probabilities of moving up and down are 0,5.

The unknown parameter is a_t, so the average rate in time t. It can be calculated from elementary bond prices. These prices are determined by backward recurrent procedure starting at maturity. However, it is important to find out bond prices with maturity t for each period of binomial tree. For calculation of elementary bond prices is used equation:

$$P_{t,s} = \left(1 + r_{t,s}\right)^{-1} 0{,}5 \cdot \left[P_{t+1,s+1}(n) + P_{t+1,s-1}(n).\right]. \tag{3.68}$$

The next step is the calibration. It is the procedure of adapting rates to ensure that the calibrated price of each elementary bond will be equal to the relevant market price for the time when the decision is made: $P'_{00}(n) = P_{00}(n)$. We continue with calculation of conditional implicit forward rates:

$$f_{t-1,t} = \frac{P_0(t-1)}{P_0(t)} - 1\,. \tag{3.69}$$

The next step is to calculate the intrinsic value of the bond for the call option:

$$IV_{t,s} = max(B_{t,s} - X;0), \tag{3.70}$$

and for the put option

$$IV_{t,s} = max(X - B_{t,s};0). \tag{3.71}$$

Finally, we can determine the price of the bond by the backward recursion starting at the maturity time. First, we set the price of the option at maturity ($F_{T,s}$) to be equal to its intrinsic value $(IV_{T,s})$, $(F_{T,s}) = (IV_{T,s})$. Then we calculate the price of the European put option recurrently by the backward procedure as he present value of the mean value for the next time moment:

$$F_{t,s} = (1 + r_{t,s})^{-1} \cdot 0{,}5 \cdot (F^{u}_{t+1,s+1} + F^{u}_{t+1,s-1}). \tag{3.72}$$

3.2.3.2. Trinomial model

Trinomial trees can be used as an alternative to binomial trees. This model is more convenient to use, because it describes better the character of short rates. The main advantage of trinomial tree is that provides an extra degree of freedom, making it easier for the tree to represent features o the interest rate process as mean reversion.

Hull and White define alternative branching methods in a trinomial tree:

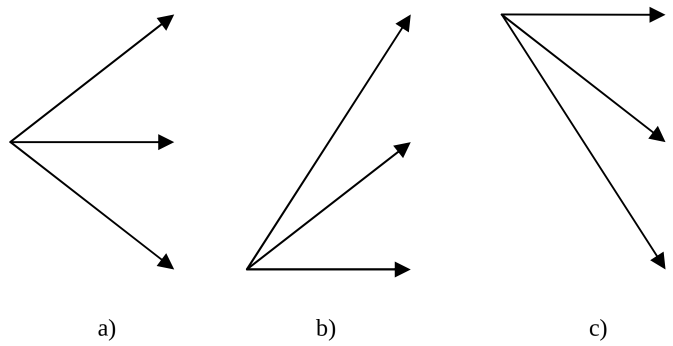

Chart 3.7 Branching methods in a trinomial tree

The usual branching is shown in chart a). It is up one/straight along/down one. Another alternative is up two/up one/straight along, as shown in chart b). This proves useful for incorporating mean reversion when interest rates are very low. A third branching method shown in chart c) is straight along/down one/down two. This is useful for incorporating mean reversion when interest rates are very high.

Hull and White published in 1994 a two-stage procedure for constructing trinomial tees to represent a wide range of one-factor models. This procedure can be used for the Hull-White model or it can be also extended to represent other models.

First stage

The Hull-White model for the instantaneous short rate r is:

$$dr = [\theta(t) - ar]dt + \sigma dz\ . \tag{3.73}$$

The first stage is based on construct a tree for variable r^* that is initially zero and then follows the process:

$$dr^* = -ar^* dt + \sigma dz\ . \tag{3.74}$$

This process is symmetrical for $r^* = 0$. The variable $r^*(t + \Delta t) - r^*(t)$ is normally distributed. If we suppose that Δt tends to zero, then mean value $r^*(t + \Delta t) - r^*(t)$ can be defined as $-ar^*(t)\Delta t$ and the variance $\sigma^2 \Delta t$.

The spacing between particular nodes in the tree is defined as:

$$\Delta r = \sigma\sqrt{3\Delta t}\ . \tag{3.75}$$

This proves to be a good choice of Δr from the viewpoint of error minimization.

The next step of this stage is to resolve which of three branching methods shown in figure 3.7 will apply at each node. This will determine the overall geometry of the tree. Once this is done, the branching probabilities must be also calculated.

We define (i, j) as the node, where the variable i is a positive integer and j is a positive or negative integer. The branching method used, must lead to probabilities on all three branches being positive. If it is not true, it is necessary to change the branching method. When $a > 0$, it is necessary to switch from the branching in figure 3.7 a) to the branching in figure 3.7 c) for a sufficiently large j . On the contrary, for sufficient small j is needed to switch to figure 3.7 b). Mostly, it is used branching shown in figure 3.7 a).

For figure 3.7 a) is set:

$$p_u \Delta r - p_d \Delta r = -aj\Delta r\Delta t\ , \tag{3.76}$$

$$p_u \Delta r^2 + p_d \Delta r^2 = \sigma^2 \Delta t + a^2 j^2 \Delta r^2 \Delta t^2\ , \tag{3.77}$$

$$p_u + p_m + p_d = 1\ . \tag{3.78}$$

Using the $\Delta r = \sigma\sqrt{3\Delta t}$, the solution to this equation is:

$$p_u = \frac{1}{6} + \frac{a^2 j^2 \Delta t \ \ - aj\Delta t}{2}\ , \tag{3.79}$$

$$p_m = \frac{2}{3} - a\ \ j^2 \Delta t^2\ , \tag{3.80}$$

$$p_d = \frac{1}{6} + \frac{a^2 j^2 \Delta t^2 + aj\Delta t}{2}\ . \tag{3.81}$$

Similarly, if the branching follows the figure 3.7 b), the probabilities are:

$$p_u = \frac{1}{6} + \frac{a^2 j^2 \Delta t \quad - aj\Delta t}{2}, \tag{3.82}$$

$$p_m = -\frac{1}{3} - a \quad j^2 \Delta t^2 - 2aj\Delta t, \tag{3.83}$$

$$p_d = \frac{7}{6} + \frac{a^2 j^2 \Delta t^2 + 3aj\Delta t}{2}. \tag{3.84}$$

Finally, if the branching follows the figure 3.7 c), the probabilities are:

$$p_u = \frac{7}{6} + \frac{a^2 j^2 \Delta t^2 + 3aj\Delta t}{2}, \tag{3.85}$$

$$p_m = -\frac{1}{3} - a \quad j^2 \Delta t^2 - 2aj\Delta t, \tag{3.86}$$

$$p_d = \frac{1}{6} + \frac{a^2 j^2 \Delta t \quad - aj\Delta t}{2}. \tag{3.87}$$

Second stage

The second stage is based on conversion the tree for r^* into a tree for r. This is accomplished by displacing the nodes on the r^*-tree so that the initial term structure is exactly matched. We define:

$$\alpha(t) = r(t) - r^*(t), \tag{3.88}$$

We must find out the value of parameter α. If we involve this equation to determination of evolution of short rate according to the Hull-white model, so:

$$dr = [(\theta(t) - ar]dt + \sigma dz, \tag{3.89}$$

and $dr^* = -ar^* dt + \sigma dz$, it follows that:

$$d\alpha = [\theta(t) - a\alpha(t)]dt. \tag{3.90}$$

Then the solution to this is:

$$\alpha(t) = F(0,t) + \frac{\sigma^2}{2a^2}(1 - e^{-at})^2. \tag{3.91}$$

This expression provides the exact relationship between r^* and r, but the r-tree determined in such way is not equal to the initial yield curve. It is better to use for determination the parameter α the iteration method, which is based on calculation of the elementary bond prices. The elementary bond price in the trinomial model is determined:

$$Q_0(n+1, j) = \sum_k p(k, j) \cdot Q_0(n,k) \cdot e^{-r_{\alpha,k} \cdot \Delta t}), \tag{3.92}$$

where $p(k, j)$ is the probability of switching from node (n,k) to node $(n+1, j)$.

When we summarize the elementary prices in time n in the particular node, we obtain current value of discounted bonds with the face value 1 m.u. and with the maturity in time n.

$$P_0(n) = \sum_{j=j_{max}}^{j_{max}} Q_0(n,j). \tag{3.93}$$

By comparison the current prices of discounted bonds with prices calculated with model, we determine the parameter α for each period. The main aim is to set this equality:

$$P_0^{mod\,el}(n) = P_0^{market}(n). \tag{3.94}$$

The figures from the 3rd chapter are overtaken from John Hull (1997) and John Hull (2005).

4. The models verification of interest rate options valuation

This part of diploma thesis is focused on verification of the models, which are described in previous chapters. We will use as an example to show how it works in practice, the zero-coupon bond EIB 0,00/23. The option valuation on the EIB 0,00/23 will be realized by using the Black's model and also using the other models (the Vasicek model, The Cox, Ingersoll, and Ross model, the Ho-Lee model, the Hull-White model and the Black-Derman-Toy model), but they have to respect statistically significance.

4.1 The input data

We assume a call option on the bond with following parameters to value the option. The underlying asset is the zero-coupon bond EIB 0,00/23. The input data of this security is possible to find out on the webpage of the Prague Stock Exchange[2]. The maturity of the bond is set on the date 20.2.2023. It is the zero-coupon bond, so there is no coupon bearing. The face value is 25 000 CZK and the current market price on the date 31.3.2009 is 40,98 %. The strike (exercise) price is determined in the amount of 20 000 CZK. The option maturity date is set on the date 1.1.2010 (T = 268/360). The valuation of all assets is performed on the date 31.3.2009.

We also need the historical rows of reference interest rate for the construction of each model (except Black's model). The aim is to verify the models so that is why we have chosen four reference interest rates (1D PRIBOR, 1W PRIBOR, 14D PRIBOR and 1M PRIBOR). These data was uploaded from the webpage the Czech National Bank[3] (see the enclosure 1)

4.2 The construction of yield curves

Before options valuation on bonds, it is necessary to construct yield curve on the settled date of valuation. For this issue we will use the procedure described in chapter 3.1.2.

The construction of the yield curve is based on the transition matrices, which show us the change of the initial rating during the exact risk horizon. For the elaboration of the diploma thesis and also the most widely used are the matrices made up by the agencies S&P or Moody's shown in table 4.1. Even if they are based on foreigner researches about the company's rating, we can use them also in Czech conditions, because the companies are evaluated according the same parameters.

[2] www.pse.cz
[3] www.cnb.cz

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	90,81%	8,33%	0,68%	0,06%	0,12%	0,00%	0,00%	0,00%
AA	0,70%	90,65%	7,79%	0,64%	0,06%	0,14%	0,02%	0,00%
A	0,09%	2,27%	91,05%	5,52%	0,74%	0,26%	0,01%	0,06%
BBB	0,02%	0,33%	5,95%	86,93%	5,30%	1,17%	0,12%	0,18%
BB	0,03%	0,14%	0,67%	7,73%	80,53%	8,84%	1,00%	1,06%
B	0,00%	0,11%	0,24%	0,43%	6,48%	83,46%	4,07%	5,20%
CCC	0,22%	0,00%	0,22%	1,30%	2,38%	11,24%	64,86%	19,79%

Table 4.1 The transition matrix: The likelihood of a credit migration to any other rating at the risk horizon (%)[4]

Known the basic transition matrix showed in table 4.1, we can continue according the figures 3.5 – 3.7 and we will get the transition matrices for years 2009 – 2023 (see the enclosure 2) The matrices are constructed till the year 2023, because our zero-coupon bond matures on the date 20.2.2023.

The following step is to find out the risk free rate for each year. We use the reference interest rate the 1Y PRIBOR, which was set down by the Czech National Bank[5] on the date 31.3.2009. Then we go on according the figure 3.8 – 3.11 to find out the spot interest rates for each year and each rating category. Our main goal is to construct the spot and forward yield curve, which can be applicable in Czech conditions and will show us the curve constructed from the Czech state bonds. State bonds are the less risky bonds at the financial markets and they are issued by Ministry of Finance of the Czech Republic. The Czech Republic is assessed according the rating agency Standard & Poor’s by rating A. Due to this assumptions we use for the forward interest rate determination the spot interest rates in the line of rating A (see the enclosure 3). This calculation is easily to count following the figure 3.12.

The outcome of the whole calculation of the spot and forward yield curve is digestedly shown in table 4.2 and chart 4.1.

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[4] Standard&Poor´s CreditWeek

[5] www.cnb.cz

Year	Spot interest rate	Forward interest rate
2009	2,84%	2,84%
2010	3,13%	3,42%
2011	3,42%	4,01%
2012	3,72%	4,61%
2013	4,01%	5,21%
2014	4,31%	5,83%
2015	4,62%	6,45%
2016	4,92%	7,08%
2017	5,23%	7,72%
2018	5,54%	8,37%
2019	5,85%	9,03%
2020	6,17%	9,69%
2021	6,48%	10,35%
2022	6,80%	11,02%

Table 4.2 The spot and the forward interest rates for each year

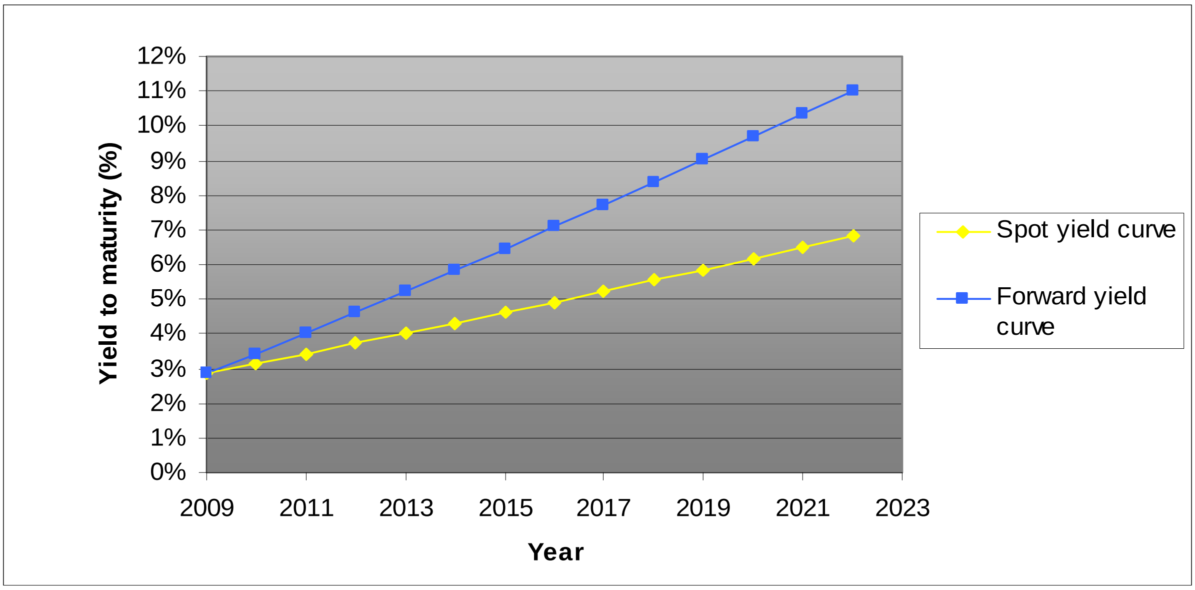

Chart 4.1 The spot and the forward yield curve

We can conclude according to the picture that the spot yield curve which we constructed is upward sloping. It means that the forward interest rate for each maturity is higher than the spot interest rate.

4.3 Using the Black's model to price the interest rate options

The Black's model is the way how to evaluate the option analytically. The principle of valuation using the Black's model was described in the chapter 3.2. The input data about the option and the underlying asset are shown in the chapter 4.1. The valuation is performed on the date 31.3.2009.

If we compare the valuation using the Black's model, we have to notice that it differs from the other models, which will be used in practical part furthermore. The whole model is based on the historical and current market prices of the bond EIB 0,00/23 and there is no influence of the reference interest rates. That is why the valuation is shown separately.

The Black's model is probably the most widely and the most simple used model to price interest rate options, but it can also brings some difficulties in the calculation, especially when the bond maturity compared with the option maturity is longer (it can be our case). But there is no strict given rule, how to predict, if the model have to face these difficulties or not, so we have to try to calculate it anymore.

Firstly, it is necessary to determine unknown input parameters. It means that in case of the Black's model we are looking only for one parameter – σ. This parameter is possible to calculate using the relation:

$$\sigma = \frac{\sigma_{\ln V_T}}{\sqrt{T}} . \tag{4.1}$$

The initial data are market prices of the state bond EIB 0,00/23 from 31.3.2008 till 31.3.2009 and they are overtaken from the Prague stock Exchange[6]. Using these data we determine the daily volatility, which is subsequently transfer to volatility of logarithm of the bond price at maturity. Know this outcome, we are able to figure out the parameter σ using:

$$\sigma = \frac{\sigma_{\ln V}^{daily} \cdot \sqrt{time\ to\ maturity\ of\ the\ option\left(daily\right)}}{\sqrt{\left(time\ to\ maturity\ of\ the\ option\left(daily\right)/\ 360\right)}} . \tag{4.2}$$

Next step is to determine forward bond price $\left(F_T\right)$, which is calculated from the relation:

$$F_T = \left(B_0 - I_0\right) \cdot e^{r \cdot T} , \tag{4.3}$$

where $I_0 = coupon \cdot \sum_{i}^{T} e^{-r_i \cdot s_i}$. (4.4)

B_0 is today's spot bond price (found out from actual bond rate at Prague Stock Exchange plus accrued interest), I_0 present value of the coupons that will be paid during the life of the option and r the interest rate for maturity T, r_i spot interest rate for period

[6] www.pse.cz

$(0,s_i)$ (found out from actual spot yield curve) and s_i maturity of i-coupon paid off during the option validity (it means in the interval $(0,T)$).

Now we have all data needed for pricing the call option. For the determination we use these relations:

$$c = e^{-r \cdot T} \cdot [F_T \cdot N(d_1) - X \cdot N(d_2)], \tag{4.5}$$

where $$d_1 = \frac{ln\left(\frac{F_T}{X}\right) + \sigma^2 \cdot \frac{T}{2}}{\sigma \cdot \sqrt{T}} \tag{4.6}$$

and $$d_2 = \frac{ln\left(\frac{F_T}{X}\right) - \sigma^2 \cdot \frac{T}{2}}{\sigma \cdot \sqrt{T}} = d_1 - \sigma \cdot \sqrt{T}\ . \tag{4.7}$$

Solution

The values of the particular sub calculations of the call option price are displayed in table 4.3. In our case of zero-coupon bearing bond the value of the forward bond price (F_T) is dependent only on the today's spot bond price (B_0).

σ	0,03537121
T	268/360
s_{coupon}	-
F_V	25000
X	20000
B_0	10245
coupon	0
I_0	0
r 268/360	0,06705787
F_T	10769,4201
d_1	-20,268091
d_2	-20,29861
N(d1)	1,2301E-91
N(d2)	6,6137E-92
c	1,89E-90

Table 4.3 The calculation of the call option price on the bond EIB 0,00/23

$$d_1 = \frac{ln\left(\frac{10769{,}4201}{20000}\right) + 0{,}0354^2 \cdot \frac{268/360}{2}}{0{,}0354 \cdot \sqrt{268/360}},$$

$$d_2 = d_1 - 0{,}0354 \cdot \sqrt{268/360},$$

$$c = e^{-0,0671 \cdot 268/360} \cdot \left[\left(10769,24201 \cdot 1,2301E{-91} \right) - \left(20000 \cdot 6,6137E{-92} \right) \right].$$

We can close up according the table 4.3 that the call option price on the bond EIB 0,00/23 is totally unrealistic. We can agree with some experts whose assumptions of inconvenience of Black's model used for some interest rate options valuation.

The Black's model assumes that the uncertainty (variance) of the underlying asset increases linearly with time to maturity. Pricing European bond options using this approach should thus be limited to options with the short time to maturity relative to the time to maturity of the bond. A rule of thumb used by some traders is that the time to maturity of the option should be no longer than one-fifth of the time to maturity on the underlying bond, Espen Gaarder Haug (1997).

4.4 Using the models of the short rate to price the interest rate options

The models of the short rate reject the assumption of lognormal distribution of the underlying asset and respect the fact that the underlying asset development is influenced by the evolution of reference interest rates in the economy.

For elaboration of the diploma thesis we have chosen four reference interest rates - 1D PRIBOR, 1W PRIBOR, 14D PRIBOR and 1M PRIBOR. These data was uploaded from the webpage the Czech National Bank[7]. The historical row of each reference interest rate consists of the exact rates from the 1.1.2009 till the 31.3.2009. The verification will be done using the Vasicek model, The Cox, Ingersoll, and Ross model, the Ho-Lee model, the Hull-White model and the Black-Derman-Toy model.

In general, the next step is to utilize the function Regression in MS Excel that provides us some of the unknown parameters. How to calculate them will be shown on the **Vasicek, the Cox, Ingersoll, and Ross, the Ho-Lee and the Hull-White model** using the historical row of the 1D PRIBOR, Zdeněk Zmeškal (2004). Even if the procedure is more or less similar for all the others models.

Firstly, we have to determine the three parameters (a, b, σ), which are included in the **Vasicek model**. The parameter a is the velocity that the short rate reverts to the long-term interest rate. The parameter b expresses the interest rate r, to which the economy tends to revert, it means the interest rate of long-term equilibrium and the parameter σ is the volatility of the short rate.

[7] www.cnb.cz

1D PRIBOR in the period from 1.1.2009 to 31.3.2009 was chosen as the short-term rate. The parameters a,b,σ are figured out by regression by the least square method. The general estimated model for interest rate is:

$$\Delta r = \Delta \hat{r} + \varepsilon. \tag{4.8}$$

The estimated discrete mean-reversion Vasicek model is then:

$$\Delta r = \Delta \hat{r} + \varepsilon = a \cdot (b - r_{t-1}) \cdot \Delta t + \hat{\sigma} \cdot \sqrt{\Delta t} \cdot \tilde{z}\ , \tag{4.9}$$

where a,b are estimated parameters, $\hat{\sigma}$ is the standard deviation, Δt is a time interval, z is a random variable, $z \in (0;1)$.

This form is necessary to transform into the linear model:

$$\Delta r = \hat{\alpha} + \hat{\beta} \cdot r_{t-1} + \varepsilon\ , \tag{4.10}$$

where

$$\hat{\alpha} = a \cdot b \cdot \Delta t\ , \tag{4.11}$$

$$\hat{\beta} = a \cdot \Delta t\ . \tag{4.12}$$

The parameters $\hat{\alpha}$ and $\hat{\beta}$ are calculated by the function Regression in MS Excel with using this relation:

$$\min \sum_t \varepsilon_t^2\ , \tag{4.13}$$

where $\varepsilon_t = \Delta r - \Delta \hat{r} = \Delta r - \left(\hat{\alpha} + \hat{\beta} \cdot r_{t-1}\right).$ (4.14)

The initial parameters of the Vasicek model are calculated on the basis of parameters which were estimated for the linear model:

$$a = -\frac{\hat{\beta}}{\Delta t}\ , \tag{4.15}$$

$$b = \frac{\hat{\alpha}/\hat{a}}{\Delta t}\ , \tag{4.16}$$

$$\sigma = \frac{\hat{\sigma}}{\sqrt{\Delta t}} = \frac{\sqrt{\frac{1}{N}\sum \varepsilon_t^2}}{\sqrt{\Delta t}}\ . \tag{4.17}$$

The Cox, Ingersoll, and Ross is estimated quite similarly. The input data and also unknown parameters are the same as in the Vasicek model. The only difference is in the figure 4.9 and 4.17.

So the estimated discrete mean-reversion CIR model is then:

$$\Delta r = \Delta \hat{r} + \varepsilon = a \cdot (b - r_{t-1}) \cdot \Delta t + \hat{\sigma} \cdot \sqrt{\Delta t \cdot r_{t-1}} \cdot \tilde{z}\ , \tag{4.18}$$

and the figure 4.17 is substituted thus:

$$\sigma = \frac{\hat{\sigma}}{\sqrt{\Delta t}} = \frac{\sqrt{\frac{1}{N}\sum \varepsilon_t^2}}{\sqrt{\Delta t \cdot r_{t-1}}} . \tag{4.19}$$

In case of the **Ho-Lee model**, we will substitute the figure 4.9 thus:

$$\Delta r = \Delta \hat{r} + \varepsilon = \theta(t) \cdot \Delta t + \hat{\sigma} \cdot \sqrt{\Delta t} \cdot \tilde{z} . \tag{4.20}$$

How to calculate the parameter $\theta(t)$ is described in the chapter 3.2.2.5 and the volatility of the short rate follows the procedure of Vasicek model (see figure 4.17).

The **Hull-White model** proposed the extension of the Vasicek and the Ho-Lee model (that is why it follows the calculation of parameters a and $\theta(t)$), so the only figure that we can substitute is figure 4.9:

$$\Delta r = \Delta \hat{r} + \varepsilon = (\theta(t) - a \cdot r_{t-1}) \cdot \Delta t + \hat{\sigma} \cdot \sqrt{\Delta t} \cdot \tilde{z} . \tag{4.21}$$

Now we have found out the unknown parameters and we can continue with estimating future evolution of the interest rate according to the individual figures used in models. Then we have to make out according to the function Regression, if the model is statistically significant. This means that the $P - value$ has to be less than 5 %. If this condition is not fulfilled, the model will be not applicable for next procedure of option valuation. The *P values* are shown in the table 4.4, the green colour means that the model for given interest rate is statistically significant and we can start with the valuation the bond option EIB 0,00/23. The unknown parameters of the statistically significant models used for the valuation are sort out in the table 4.5.

Reference interest rate	Parameter	Model				
		Vasicek	CIR	Ho-Lee	Hull-White	BDT
1D PRIBOR	$\hat{\alpha}$	0,8247	0,0432	0,9727	6,47E-08	0,7534
	$\hat{\beta}$	0,0453	0,0307	0,7922	3,65E-09	0,7551
1W PRIBOR	$\hat{\alpha}$	0,7147	0,0278	2,70E-06	6,49E-05	0,2354
	$\hat{\beta}$	0,1083	0,0227	0,5293	0,0126	0,3001
14D PRIBOR	$\hat{\alpha}$	0,9873	0,0191	8,65E-09	1,89E-10	0,7283
	$\hat{\beta}$	0,9214	0,0168	0,0472	6,08E-06	4,23E-79
1M PRIBOR	$\hat{\alpha}$	0,7520	0,0710	9,45E-09	0,0001	0,4193
	$\hat{\beta}$	0,2241	0,0755	0,2288	0,0035	5,90E-60

Table 4.4 P-values for individual models and different reference interest rates

Reference interest rate	CIR		Ho-Lee		Hull-White	
	Parameter	Value	Parameter	Value	Parameter	Value
1D PRIBOR	a	6,5055			a	36,9348
	b	0,1900			σ	0,0637
	σ	0,1446			$\theta(t)$	0,4327
1W PRIBOR	a	4,7669			a	0,9316
	b	0,0163			σ	0,0151
	σ	0,0139				0,0456
14D PRIBOR	a	2,3840			a	0,7025
	b	0,0178	σ	0,0121	σ	0,1100
	σ	0,0433	$\theta(t)$	0,0210	$\theta(t)$	0,0454
1M PRIBOR					a	0,4720
					σ	0,0119
					$\theta(t)$	0,0309

Table 4.5 The unknown parameters of the statistically significant models used for the valuation the bond option EIB 0,00/23

4.4.1 Using the models of the short rate to price the interest rate options with reference interest rate 1D PRIBOR

Firstly, we have chosen as an input data the reference interest rate 1D PRIBOR. We follow the general procedure described in the chapter 4.4. According to the outcome of the regression showed in table 4.4, we can determine which models will be used for the future valuation of the bond option EIB 0,00/23 in the case of 1D PRIBOR. So we will use the Cox, Ingersoll, and Ross model and the Hull-White model.

4.4.1.1. Using the Cox, Ingersoll, and Ross model to price the interest rate options

The principle of valuation using the Cox, Ingersoll, and Ross model was described in the chapter 3.2.2.4.

Even if the base is the same as in the Vasicek model, the CIR model has one big advantage – it relieved the shortage of the negative short-term interest rate. The process of the short term interest rate is described thus:

$$dr = a(b-r)dt + \sigma\sqrt{r} \cdot dz\ . \tag{4.18}$$

The first step to start calculation is to determine the three parameters (a,b,σ). The parameter a is the velocity that the short rate reverts to the long-term interest rate. The parameter b expresses the interest rate r, to which the economy tends to revert, it means the interest rate of long-term equilibrium and the parameter σ is the volatility of the short rate.

These parameters are possible to calculate with Regression by the least square method. Then we can continue with pricing the interest rate option on the bond EIB 0,00/23.

Solution

Firstly, we have to find out the three parameters (a,b,σ). Their values are displayed in table 4.6.

a	6,5055
b	0,1900
σ	0,1446

Table 4.6 The input data of the CIR model

The following step is to price the option on the bond EIB 0,00/23. For this calculation we use the formulas (3.34) – (3.37). We will apply these input data – face value is 25 000 CZK, strike price is 20 000 CZK, the option maturity is 268/360 and the bond maturity is 13+317/360.

$$\gamma = \sqrt{6{,}5055^2 + 2 \cdot 0{,}1446^2} = 6{,}5079$$

$$B(0{,}268/360) = \frac{2 \cdot (e^{\gamma(268/360-0)} - 1)}{(\gamma + 6{,}5055)(e^{\gamma(268/360-0)} - 1) + 2\gamma} = 0{,}1525$$

$$B(0{,}13\frac{317}{360}) = \frac{2 \cdot (e^{\gamma(13\frac{317}{360}-0)} - 1)}{(\gamma + 8{,}2577)(e^{\gamma(13\frac{317}{360}-0)} - 1) + 2\gamma} = 0{,}1537$$

$$A(0{,}268/360) = 0{,}9990$$

$$A(0{,}13\frac{317}{360}) = 0{,}9782$$

$$P(0{,}268/360) = A(0{,}268/360)e^{-B(0{,}268/360) \cdot 0{,}0181} = 0{,}9963$$

$$P(0{,}13\frac{317}{360}) = A(0{,}13\frac{317}{360})e^{-B(0{,}13\frac{317}{360}) \cdot 0{,}0181} = 0{,}9755$$

$\sigma_p = 0{,}0062$

$$h = \frac{1}{\sigma_p} ln \frac{25000 \cdot P(0{,}13\frac{317}{360})}{20000 \cdot P(0{,}268/360)} + \frac{\sigma_p}{2} = 32{,}7927$$

$$c = 25000 \cdot P(0{,}13\frac{317}{360})N(h) - 20000 \cdot P(0{,}268/360)N(h - \sigma_p) = 4461{,}06$$

According to the Cox, Ingersoll, and Ross model, where the input data is 1D PRIBOR, the price of the call option on the bond EIB 0,00/23 is 4461,06 CZK. This is the price for the right to buy one piece of the bond EIB 0,00/23 for the strike price 20000 CZK.

4.4.1.2. Using the Hull-White model to price the interest rate options

The principle of valuation using the Hull-White model was described in the chapter 3.2.2.6.

The first step is to determine the parameters a and σ. These parameters are also calculated using the function Regression in the MS Excel. The process of the short term interest rate is described thus:

$$dr = (\theta(t) - ar)dt + \sigma dz \tag{4.19}$$

There is also another unknown parameter in relation above. The parameter $\theta(t)$ can be calculated from the initial yield curve:

$$\theta(t) = F_t(0,t) + aF(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at}) \tag{4.20}$$

Now we know all the parameters and we can begin to price the option on the bond EIB 0,00/23.

Solution

The unknown parameters are shown in the table 4.7. The procedure how to calculate them is mentioned above.

a	36,9348
σ	0,0637
θ(t)	0,4327

Table 4.7 The input data of the Hull-White model

In this moment, we can start with pricing the call option on the bond EIB 0,00/23. We use the formulas (3.51) – (3.57) and we are counting with input data - face value is

25 000 CZK, strike price is 20 000 CZK, the option maturity is 268/360 and the bond maturity is 13+317/360.

$$B(0{,}268/360;0{,}13\frac{317}{360}) = \frac{1-e^{-36{,}9348(13\frac{317}{360}-268/360)}}{36{,}9348} = 0{,}0271$$

$$\ln A(0{,}268/360;0{,}13\frac{317}{360}) = -0{,}0276$$

$$P(0{,}268/360) = 0{,}9597$$

$$P(0{,}13\frac{317}{360}) = 0{,}9723$$

$$\sigma_p = 0{,}0002$$

$$h = \frac{1}{\sigma_P}\ln\frac{25000\cdot P(0{,}13\frac{317}{360})}{20000\cdot P(0{,}268/360)} + \frac{\sigma_P}{2} = 3{,}1487$$

$$c = 25000\cdot P(0{,}13\frac{317}{360})N(h) - 20000\cdot P(0{,}268/360)N(h-\sigma_P) = 5108{,}14$$

We calculate the price of the option on the bond EIB 0,00/23 using The Hull-White model and the input data 1D PRIBOR and the result of this calculation is 5108,14 CZK.

4.4.2 Using the models of the short rate to price the interest rate options with reference interest rate 1W PRIBOR

The historical row of the reference interest rate 1W PRIBOR was chosen as the second example. We found out that we can use again only two models – the Cox, Ingersoll, and Ross and the Hull-White model. The others are not statistically significant.

4.4.2.1. Using the Cox, Ingersoll, and Ross model to price the interest rate options

Since the calculation procedure is the same as in the chapter 4.4.1.1., we will illustrate only the solution.

Solution

Firstly, we were looking for the three unknown parameters (a,b,σ). Their values are showed in table 4.8.

a	4,7669
b	0,0163
σ	0,0139

Table 4.8 The input data of the CIR model

The following steps and the input data of the zero-coupon bond are the same as in the chapter 4.4.1.1.

$$\gamma = \sqrt{4{,}7669^2 + 2 \cdot 0{,}0139^2} = 4{,}7669$$

$$B(0{,}268/360) = \frac{2 \cdot (e^{\gamma(268/360-0)} - 1)}{(\gamma + 4{,}7669)(e^{\gamma(268/360-0)} - 1) + 2\gamma} = 0{,}2037$$

$$B(0{,}13\frac{317}{360}) = \frac{2 \cdot (e^{\gamma(13\frac{317}{360}-0)} - 1)}{(\gamma + 4{,}7669)(e^{\gamma(13\frac{317}{360}-0)} - 1) + 2\gamma} = 0{,}2097$$

$$A(0{,}268/360) = 0{,}9999$$

$$A(0{,}13\frac{317}{360}) = 0{,}9997$$

$$P(0{,}268/360) = A(0{,}268/360)e^{-B(0{,}268/360)\cdot 0{,}0191} = 0{,}9961$$

$$P(0{,}13\frac{317}{360}) = A(0{,}13\frac{317}{360})e^{-B(0{,}13\frac{317}{360})\cdot 0{,}0191} = 0{,}9957$$

$$\sigma_p = 0{,}0009$$

$$h = \frac{1}{\sigma_p} ln \frac{25000 \cdot P(0{,}13\frac{317}{360})}{20000 \cdot P(0{,}268/360)} + \frac{\sigma_p}{2} = 235{,}4251$$

$$c = 25000 \cdot P(0{,}13\frac{317}{360})N(h) - 20000 \cdot P(0{,}268/360)N(h - \sigma_p) = 4971$$

The price of the call option on the bond EIB 0,00/23 calculated using the Cox, Ingersoll, and Ross model is 4971 CZK. This is the price, which we are willing to pay on the date 31.3.2009 for the right to buy the bond EIB 0,00/23 for the strike price 20000 CZK on the date 1.1.2010.

4.4.2.2. Using the Hull-White model to price the interest rate options

The principle of valuation using the Hull-White model was described in the chapter 3.3.6 and 4.4.1.2, so we will directly overcome to solution.

Solution

The data required for the calculation are shown in the table 4.9. and the option price is calculated underneath.

a	0,9316
σ	0,0151
□(t)	0,0456

Figure 4.9 The input data of the Hull-White model

$$B(0{,}268/360; 0{,}13\frac{317}{360}) = \frac{1 - e^{-0{,}9316(13\frac{317}{360} - 268/360)}}{0{,}9316} = 0{,}6506$$

$$\ln A(0{,}268/360; 0{,}13\frac{317}{360}) = -0{,}0204$$

$$P(0{,}268/360) = 0{,}9712$$

$$P(0{,}13\frac{317}{360}) = 0{,}9723$$

$$\sigma_p = 0{,}0103$$

$$h = \frac{1}{\sigma_P} \ln \frac{25000 \cdot P(0{,}13\frac{317}{360})}{20000 \cdot P(0{,}268/360)} + \frac{\sigma_P}{2} = 21{,}8631$$

$$c = 25000 \cdot P(0{,}13\frac{317}{360})N(h) - 20000 \cdot P(0{,}268/360)N(h - \sigma_P) = 4883{,}04$$

The result of our option price calculation using the Hull-White model and the historical row of 1W PRIBOR is 4883,04 CZK.

4.4.3 Using the models of the short rate to price the interest rate options with reference interest rate 14D PRIBOR

From the table 4.4 we can easily recognize that in the column of the 14D PRIBOR, which was chosen as the third example, are statistically significant three models - the Cox, Ingersoll, and Ross, the Ho-Lee and the Hull-White model.

4.4.3.1. Using the Cox, Ingersoll, and Ross model to price the interest rate options

The description how to proceed in calculation is shown in the chapter 4.4.1.1.

Solution

The values of unknown parameters (a, b, σ) are put down in the table 4.10.

a	2,3840
b	0,0178
σ	0,0433

Table 4.10 The input data of the CIR model

The following steps and the input data of the zero-coupon bond are the same as in the chapter 4.4.1.1.

$$\gamma = \sqrt{2{,}384^2 + 2 \cdot 0{,}0433^2} = 2{,}384$$

$$B(0{,}268/360) = \frac{2 \cdot (e^{\gamma(268/360-0)} - 1)}{(\gamma + 2{,}3840)(e^{\gamma(268/360-0)} - 1) + 2\gamma} = 0{,}3484$$

$$B(0{,}13\frac{317}{360}) = \frac{2 \cdot (e^{\gamma(13\frac{317}{360}-0)} - 1)}{(\gamma + 4{,}7669)(e^{\gamma(13\frac{317}{360}-0)} - 1) + 2\gamma} = 0{,}4195$$

$$A(0{,}268/360) = 0{,}9999$$

$$A(0{,}13\frac{317}{360}) = 0{,}9999$$

$$P(0{,}268/360) = A(0{,}268/360)e^{-B(0{,}268/360)\cdot 0{,}0197} = 0{,}9932$$

$$P(0{,}13\frac{317}{360}) = A(0{,}13\frac{317}{360})e^{-B(0{,}13\frac{317}{360})\cdot 0{,}0197} = 0{,}9917$$

$$\sigma_p = 0{,}0008$$

$$h = \frac{1}{\sigma_p} ln \frac{25000 \cdot P(0{,}13\frac{317}{360})}{20000 \cdot P(0{,}268/360)} + \frac{\sigma_p}{2} = 270{,}572$$

$$c = 25000 \cdot P(0{,}13\frac{317}{360})N(h) - 20000 \cdot P(0{,}268/360)N(h - \sigma_p) = 4929{,}77$$

The price of the call option on the bond EIB 0,00/23 calculated using the CIR model and the 14D PRIBOR is 4929,77 CZK.

4.4.3.2. Using the Ho-Lee model to price the interest rate options

The principle of valuation using the Ho-Lee model was described in the chapter 3.2.2.5.

If we compare the Ho-Lee model with the Vasicek and the CIR model, it brings one advantage. The Ho-Lee model includes the variable that ensures correspondence between this model and the yield curve at the time of construction. The process of the short term interest rate is described thus:

$$dr = \theta(t)dt + \sigma dz \,. \tag{4.21}$$

The variable $\theta(t)$ defines the average direction that r moves at time t. It is also possible to use for its calculation this relation:

$$\theta(t) = F_t(0,t) + \sigma^2 t \,, \tag{4.22}$$

where $F_T(0,t)$ is instantaneous forward rate for a period $(0,t)$ and we can infer it from the current yield curve.

If we neglect the last part of the relation and suppose that $F_T(0,t)$ is equal to $\theta(t)$, then the average direction that the short rate will be moving in the future is approximately equal to the slope of the instantaneous forward curve. The next steps of the calculation procedure are quite similar to the previous models.

Solution

Using the function Regression we found out the only one unknown parameter σ in the Ho-Lee model. Its value is 0,0121. The following step is to determine the price of the call option on the bond EIB 0,00/23. This calculation is made according to the formulas 3.40 – 3.41.

$$\ln A(0{,}268/360;0{,}13\frac{317}{360}) = -0{,}0084$$

$$P(0{,}268/360) = 0{,}9772$$

$$P(0{,}13\frac{317}{360}) = 0{,}7544$$

$$\sigma_p = 0{,}1375$$

$$h = \frac{1}{\sigma_P}\ln\frac{25000 \cdot P(0{,}13\frac{317}{360})}{20000 \cdot P(0{,}268/360)} + \frac{\sigma_P}{2} = -0{,}1905$$

$$c = 25000 \cdot P(0{,}13\frac{317}{360})N(h) - 20000 \cdot P(0{,}268/360)N(h - \sigma_P) = 4765{,}07$$

The result from the calculation using the Ho-Lee model is 4765,07 CZK. This is the price we are willing to pay for the right to buy the bond EIB 0,00/23 for the strike price 20000 CZK.

4.4.3.3. Using the Hull-White model to price the interest rate options

The instruction how to calculate the bond price using the Hull-White model was described in the chapter 3.3.6 and 4.4.1.2.

Solution

The unknown parameters required for the calculation are shown in the table 4.11 and the procedure of counting the option price followed.

a	0,7025
σ	0,0110
□(t)	0,0454

Table 4.11 The input data of the Hull-White model

$$B(0{,}268/360;0{,}13\frac{317}{360}) = \frac{1-e^{-0{,}7025(13\frac{317}{360}-268/360)}}{0{,}7025} = 0{,}7184$$

$$\ln A(0{,}268/360;0{,}13\frac{317}{360}) = -0{,}0139$$

$$P(0{,}268/360) = 0{,}9718$$

$$P(0{,}13\frac{317}{360}) = 0{,}9723$$

$$\sigma_p = 0{,}0104$$

$$h = \frac{1}{\sigma_P}\ln\frac{25000\cdot P(0{,}13\frac{317}{360})}{20000\cdot P(0{,}268/360)} + \frac{\sigma_P}{2} = 3{,}1487$$

$$c = 25000\cdot P(0{,}13\frac{317}{360})N(h) - 20000\cdot P(0{,}268/360)N(h-\sigma_P) = 4868{,}04$$

The outcome of our option price calculation using the Hull-White model and the historical row of 14D PRIBOR is 4868,04 CZK.

4.4.4 Using the models of the short rate to price the interest rate options with reference interest rate 1M PRIBOR

The last reference interest rate that will be used in the diploma thesis is 1M PRIBOR. According the P-value estimation, there is only one model – the Hull-White, which can be applicable for pricing the interest rate option.

4.4.4.1. Using the Hull-White model to price the interest rate options

As the procedure was mentioned above in the chapter 3.3.6 and 4.4.1.2, we can directly continue with the solution.

Solution

The data required for the calculation are shown in the table 4.12 and the option price is calculated underneath.

a	0,4720
σ	0,0119
□(t)	0,0309

Table 4.12 The input data of the Hull-White model

$$B(0{,}268/360;0{,}13\frac{317}{360}) = \frac{1-e^{-0{,}472(13\frac{317}{360}-268/360)}}{0{,}472} = 07971$$

$$\ln A(0{,}268/360;0{,}13\frac{317}{360}) = -0{,}0102$$

$$P(0{,}268/360) = 0{,}9734$$

$$P(0{,}13\frac{317}{360}) = 0{,}9723$$

$$\sigma_p = 0{,}0186$$

$$h = \frac{1}{\sigma_P}\ln\frac{25000\cdot P(0{,}13\frac{317}{360})}{20000\cdot P(0{,}268/360)} + \frac{\sigma_P}{2} = 3{,}1487$$

$$c = 25000\cdot P(0{,}13\frac{317}{360})N(h) - 20000\cdot P(0{,}268/360)N(h-\sigma_P) = 4835{,}45$$

According to the Hull-White model, where the input data is 1M PRIBOR, the price of the call option on the bond EIB 0,00/23 is 4835,45 CZK. This is the price for the right to buy one piece of the bond EIB 0,00/23 for the strike price 20 000 CZK.

4.5 Summary

In the previous chapters we tried to verify the models used for the pricing the interest rate options. The results of the option prices on the bond EIB 0,00/23 are displayed in the table 4.13 and its standard deviations in the table 4.14. These two criteria are mentioned because we will use them for comparison of individual models.

Model	**Option price on the bond EIB 0,00/23 (CZK)**			
	1D PRIBOR	1W PRIBOR	14D PRIBOR	1M PRIBOR
Vasicek	x	x	x	x
Cox, Ingersoll, and Ross	4461,06	4971,00	4929,77	x
Ho-Lee	x	x	4765,07	x
Hull-White	5108,14	4883,04	4868,04	4835,45
Black-Derman-Toy	x	x	x	x

Table 4.13 The option price on the bond EIB 0,00/23

Model	**Standard deviation (%)**			
	1D PRIBOR	1W PRIBOR	14D PRIBOR	1M PRIBOR
Vasicek	x	x	x	x
Cox, Ingersoll, and Ross	14,456%	1,393%	0,433%	x
Ho-Lee	x	x	1,213%	x
Hull-White	6,373%	1,506%	1,101%	1,199%
Black-Derman-Toy	x	x	x	x

Table 4.14 The standard deviation of the used models

The Black's model is missing in the 4.13 and 4.14, because it is based on the totally different input data. The inputs are the historical and current market prices of the bond EIB 0,00/23 and there is no influence of the reference interest rates. This is the main disadvantage and it also leads to the impossibility to use it for American options valuation, the embedded options valuation etc. On the other hand the calculation is quite easy and fast. We have to comment according to calculation that in our case the valuation using the Black's model was unrealistic.

The models Vasicek and CIR respects an empirical fact, that interest rates regularly revert to the long-run equilibrium, but they do not take into account the actual yield curve. In addition, the Vasicek model can also contain the negative interest rates. We avoid these difficulties in valuation because the Vasicek model applying our input data was statistically insignificant. The valuation according the Cox, Ingersoll, and Ross was possible using the historical row of 1D, 1W and 14 D PRIBOR.

The non-arbitrage models, the Ho-Lee, the Hull-White and the Black-Derman-Toy are constructed to be in correspondence with the current yield curve in time of the construction. The disadvantage of the Ho-Lee model is that it does not follow the mean-reversion process. If the inputs consist of the reference interest rate 14D PRIBOR, the Ho-Lee model is in our case applicable. The shortage of the Ho-Lee model is eliminated by the Hull-White. The Hull-White model brings us the results of the option price using all the given input data.

Firstly, we have chosen for our comparison the option price on the bond EIB 0,00/23. According to our calculations is the best option price using the CIR model with input data 1D PRIBOR, followed by the Ho-Lee model with input data 14D PRIBOR and the worst using the Hull-White model with enter data 1D PRIBOR. The prices are digestedly sort out in the table 4.15.

Model	Input data	Price (CZK)
CIR	1D PRIBOR	4461,06
Ho-Lee	14D PRIBOR	4765,07
Hull-White	1M PRIBOR	4835,45
Hull-White	14D PRIBOR	4868,04
Hull-White	1W PRIBOR	4883,04
CIR	14D PRIBOR	4929,77
CIR	1W PRIBOR	4971,00
Hull-White	1D PRIBOR	5108,14

Table 4.15 The option price on the bond EIB 0,00/23 sort out from the best to the worst

Model	Input data	Standard deviation (%)
CIR	14D PRIBOR	0,433
Hull-White	14D PRIBOR	1,101
Hull-White	1M PRIBOR	1,199
Ho-Lee	14D PRIBOR	1,213
CIR	1W PRIBOR	1,393
Hull-White	1W PRIBOR	1,506
Hull-White	1D PRIBOR	6,373
CIR	1D PRIBOR	14,456

Table 4.16 The standard deviation of the used models sort out from the best to the worst

The second criterion that we used for comparison is the standard deviation. Based on the table 4.16 we can comment that the best model for valuing the option price on the bond EIB 0,00/23 is the CIR model with input data 14D PRIBOR and next is the Hull-White also with input data 14D PRIBOR.

We determined two best models for valuing the option price on the bond EIB 0,00/23 according the price and the standard deviation, but both these comparisons used separately can be misleading. The problems are caused by different input assumptions, restrictions, predicable ability of each model etc. (see chapter 3.2). These reasons lead to our final solution to choose for the option valuation on the bond EIB 0,00/23 the Hull-White model with input data 14D PRIBOR or 1M PRIBOR.

5. Conclusion

The options belong to the financial derivatives and their underlying asset can be practically anything from the commodities to another financial derivatives. Basically, options transactions are quite simple. The buyer of an option pays the option premium to the seller and then he is waiting, how the market is developing – in case of the interest rate options we can discuss about the interest rate trends. If the trend is favourable at the date of option maturity, the option will be exercised (if not, the option expires).

The main aim of the diploma thesis was to verify and compare the chosen models that are possible to use for interest rate options valuation. The aim was realized using the example of zero-coupon bearing bond.

The first chapter was simple introduction and also the adumbration of the diploma thesis.

The second chapter was firstly devoted to general characteristic of bonds. It means theirs most important features, functions, types and the description how to calculate the market price. The second part of this chapter was focused on option contracts in general (on history, basic terminology, trading and using. More deeply was characterized the issue of the interest rate options and theirs types (bond options, caps, floors, collars, swaptions and embedded interest rate options).

The following chapter included description of yield curves construction and of method, which are used for interest rate options valuation. It was explained how to valuate the derivatives using the Black's model, using the models of the short rate (The Vasicek model, the Cox, Ingersoll, and Ross model, the Ho-Lee model, the Hull-White model) and also using numerical models (binomial and trinomial models).

The fourth chapter can be named as a practical part. Firstly it was necessary to construct the spot and forward yield curves using the procedure suggested by Andrea Resti. We continued with the option valuation on the zero-coupon bearing bond EIB 0,00/23 using the Black's model, which gave us unrealistic result (see chapter 4.3). This model is based on different input data, so the calculation was shown separately. The next part of this chapter was devoted to the models of the short rate. Firstly, we have chosen four various reference interest rates (1D PRIBOR, 1W PRIBOR, 14D PRIBOR and 1M PRIBOR) and we had to find out the statistically significance of each model according to the Regression and based on these input data. We continued with option valuation on the bond EIB 0,00/23 with models that fulfilled the condition of statistically significance. For the final models evaluation were set down two criteria – the option price and the standard deviation. According the first one, we concluded that the best model to price the bond option was the Cox, Ingersoll, and Ross model with the input data 1D PRIBOR and using the second one, was the best model Cox, Ingersoll, and Ross with input data 14D PRIBOR. We remarked, the CIR model does not respect the important assumption of the actual yield curve, so we finally considered as the best solution for option valuation on the zero-coupon bearing bond EIB 0,00/23 the Hull-White model with input data 14D PRIBOR or 1M PRIBOR.

The fifth chapter gave us briefly conclusion and future view on the financial derivatives market.

The interest rate options valuation is complicated process influenced by interest rate trends, discounting, the yield curves features, volatilities etc. If we get back to the practical part and compare the particular results, we can easily recognize theirs variety. It is not always possible to say which model is the best and recommend it, because each of these models has the different initial assumptions and it is also necessary to respect the options features. The calculations of the valuation are so complicated so they are widely made without the software programs.

There are three types of traders at the markets with financial derivatives - hedgers, speculators and arbitrageurs. They use these instruments for certain purposes. In general, there are two contradictory opinions about using the options. The first is that they contribute to solve the problems with the risk management and the second; the contradictory one is that they are threats for the economy stabilization.

The options in the Czech Republic are still not very used. The tradition of these instruments is not so long and also the subconsciousness is not widespread, even though the banks are offering these products. The option trades are more convenient for the corporate clients, because the nominal values required by banks during the conclusion of the option contracts are minimally in the amount of ten millions CZK.

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List of Abbreviations

a	velocity that the short rate reverts to the long-term interest rate (b)
AI	accrued interest
$\hat{\alpha}$	parameter using in calculation of regression
B	long-term interest rate
B_0	today´s spot bond price
$\hat{\beta}$	parameter using in calculation of regression
C	value of the call option
C	coupon
CBOE	The Chicago Board Options Exchanges
CBOT	The Chicago Board of Trade
CF	cash flow
CIR	Cox, Ingersoll, Ross
CZK	Czech Koruna
dt	length of the time interval
Δt	length of the time interval
dr	interest rate shift in time
dz	specific Wiener process
$\theta(t)$	variable defines the average direction that r moves at time t (Ho-Lee model)
e.g.	exempli gratia
etc.	and so on
ε	short time interval
f_t	forward rate
$f_{t-1,t}$	implicit one-year forward rate
F_T	forward price
FV	face value
I_0	present value of the coupons that will be paid during the life of the option
LIBOR	reference interest rate
m	coupon payments per year
m.u.	money unit
N(d)	cumulative probability for normal distibution
OTC	over the counter

p	value of the put option
$P_{00}(n)$	today's market bond price
$P'_{00}(n)$	calculated bond price using the binomial or trinomial model
PRIBOR	reference interest rate
p_d	risk neutral probability between nodes (moving down)
p_u	risk neutral probability between nodes (moving up)
PV	present value
r_t	short-term interest rate at time t
s	bond time to maturity
s_t	short-term yield (forward for one period)
t	coupon time to maturity
USD	US Dolar
V_T	value of the underlying asset at time T
X	strike (exercise) price
Z	random factor from normal distribution N(0,1)
σ	volatility
$\sigma \cdot \sqrt{T}$	standard deviation
(t,s)	definition of the node (binomial and trinomial model)

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V Ostravě dne 10.7.2009

……………………………….
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Enclosures

Enclosure 1	The reference interest rates from 1.1.2009 till 31.3.2009
Enclosure 2	Transition matrices for years 2009 – 2023
Enclosure 3	The spot interest rates

Enclosure 1 The reference interest rates from 1.1.2009 till 31.3.2009

1D PRIBOR	1W PRIBOR	14D PRIBOR	1M PRIBOR
1,79%	2,43%	2,54%	3,18%
2,02%	2,43%	2,50%	3,14%
2,16%	2,42%	2,48%	3,10%
2,21%	2,42%	2,47%	3,08%
2,26%	2,44%	2,47%	3,07%
2,27%	2,44%	2,47%	3,04%
2,21%	2,44%	2,48%	3,02%
2,03%	2,44%	2,47%	2,99%
2,12%	2,41%	2,47%	2,97%
2,14%	2,40%	2,47%	2,94%
2,21%	2,38%	2,44%	2,87%
2,24%	2,39%	2,44%	2,85%
2,24%	2,39%	2,44%	2,82%
2,33%	2,41%	2,45%	2,77%
2,79%	2,46%	2,47%	2,73%
2,49%	2,47%	2,49%	2,71%
2,24%	2,46%	2,47%	2,66%
2,23%	2,45%	2,46%	2,62%
2,11%	2,42%	2,45%	2,60%
1,98%	2,38%	2,41%	2,58%
2,06%	2,35%	2,38%	2,56%
2,69%	2,32%	2,34%	2,53%
2,71%	2,29%	2,30%	2,50%
2,62%	2,26%	2,27%	2,47%
2,29%	2,17%	2,20%	2,42%
1,92%	2,01%	2,04%	2,30%
1,87%	2,01%	2,04%	2,30%
1,83%	2,00%	2,02%	2,28%
1,82%	1,98%	2,02%	2,26%
1,83%	1,96%	2,01%	2,25%
1,88%	1,98%	2,02%	2,26%
1,84%	1,98%	2,01%	2,25%
1,82%	1,97%	2,01%	2,25%
1,86%	1,98%	2,01%	2,27%
1,84%	1,97%	2,01%	2,27%
1,86%	1,98%	2,02%	2,27%
1,82%	1,98%	2,01%	2,26%
1,39%	1,91%	1,99%	2,28%
1,34%	1,87%	1,99%	2,28%
1,10%	1,88%	1,97%	2,26%
1,15%	1,89%	1,95%	2,25%
1,22%	1,87%	1,95%	2,26%
1,21%	1,89%	1,95%	2,25%
1,26%	1,90%	1,95%	2,26%
1,64%	1,89%	1,97%	2,27%
1,74%	1,90%	1,95%	2,26%
1,83%	1,89%	1,95%	2,27%
1,92%	1,90%	1,97%	2,27%
1,90%	1,89%	1,98%	2,28%
1,95%	1,90%	2,00%	2,31%
1,93%	1,91%	2,00%	2,29%
1,92%	1,93%	2,00%	2,28%
1,89%	1,97%	2,00%	2,30%
1,89%	1,97%	2,00%	2,30%
1,85%	1,97%	1,99%	2,29%
1,82%	1,98%	1,99%	2,28%
1,84%	1,98%	1,99%	2,28%
1,73%	1,96%	1,97%	2,28%
1,74%	1,97%	1,97%	2,27%

1,71%	1,96%	1,96%	2,25%
1,69%	1,94%	1,96%	2,26%
1,71%	1,93%	1,96%	2,26%
1,85%	1,92%	1,97%	2,25%
1,56%	1,91%	1,97%	2,26%
1,81%	1,91%	1,97%	2,25%

Enclosure 2 Transition matrices for years 2009 – 2023

1.year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,90810	0,08330	0,00680	0,00060	0,00120	0,00000	0,00000	0,00000
AA	0,00700	0,90650	0,07790	0,00640	0,00060	0,00140	0,00020	0,00000
A	0,00090	0,02270	0,91050	0,05520	0,00740	0,00260	0,00010	0,00060
BBB	0,00020	0,00330	0,05950	0,86930	0,05300	0,01170	0,00120	0,00180
BB	0,00030	0,00140	0,00670	0,07730	0,80530	0,08840	0,01000	0,01060
B	0,00000	0,00110	0,00240	0,00430	0,06480	0,83460	0,04070	0,05200
CCC	0,00220	0,00000	0,00220	0,01300	0,02380	0,11240	0,64860	0,19790
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

2.year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,82524	0,15131	0,01890	0,00207	0,00219	0,00025	0,00003	0,00002
AA	0,01277	0,82412	0,14198	0,01572	0,00205	0,00279	0,00039	0,00018
A	0,00181	0,04152	0,83413	0,09898	0,01581	0,00588	0,00041	0,00148
BBB	0,00045	0,00731	0,10654	0,76315	0,08998	0,02492	0,00283	0,00481
BB	0,00057	0,00293	0,01644	0,13034	0,65862	0,14701	0,01823	0,02586
B	0,00012	0,00207	0,00505	0,01300	0,10748	0,70692	0,06102	0,10415
CCC	0,00344	0,00043	0,00465	0,02218	0,04259	0,16897	0,42551	0,33238
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

3.year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,75047	0,20635	0,03475	0,00447	0,00311	0,00069	0,00009	0,00007
AA	0,01750	0,75141	0,19452	0,02696	0,00423	0,00426	0,00058	0,00053
A	0,00271	0,05708	0,76873	0,13360	0,02457	0,00974	0,00087	0,00271
BBB	0,00074	0,01176	0,14366	0,67644	0,11539	0,03828	0,00468	0,00906
BB	0,00081	0,00459	0,02776	0,16601	0,54738	0,18454	0,02455	0,04433
B	0,00030	0,00298	0,00809	0,02374	0,13454	0,60652	0,06944	0,15415
CCC	0,00408	0,00110	0,00724	0,02909	0,05659	0,19289	0,28332	0,42587
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

4.year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,68298	0,25038	0,05310	0,00782	0,00407	0,00129	0,00017	0,00019
AA	0,02134	0,68712	0,23740	0,03935	0,00704	0,00587	0,00080	0,00108
A	0,00359	0,06990	0,71253	0,16089	0,03324	0,01404	0,00146	0,00435
BBB	0,00107	0,01642	0,17284	0,60518	0,13243	0,05098	0,00658	0,01450
BB	0,00105	0,00637	0,03968	0,18930	0,46236	0,20719	0,02911	0,06491
B	0,00050	0,00384	0,01152	0,03501	0,15062	0,52620	0,07110	0,20091
CCC	0,00437	0,00189	0,00990	0,03459	0,06642	0,19819	0,19221	0,49262
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

5.year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,62202	0,28510	0,07299	0,01207	0,00514	0,00204	0,00027	0,00038
AA	0,02441	0,63019	0,27223	0,05230	0,01035	0,00765	0,00103	0,00183
A	0,00443	0,08043	0,66406	0,18229	0,04156	0,01865	0,00213	0,00644
BBB	0,00141	0,02113	0,19569	0,54627	0,14347	0,06255	0,00841	0,02105
BB	0,00127	0,00827	0,05156	0,20380	0,39679	0,21939	0,03217	0,08671
B	0,00070	0,00469	0,01530	0,04593	0,15903	0,46092	0,06908	0,24401
CCC	0,00444	0,00273	0,01259	0,03911	0,07282	0,19332	0,13344	0,54174
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

6.year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,56692	0,31196	0,09366	0,01713	0,00638	0,00291	0,00038	0,00066
AA	0,02684	0,57968	0,30033	0,06539	0,01405	0,00962	0,00130	0,00280
A	0,00524	0,08903	0,62210	0,19896	0,04936	0,02345	0,00285	0,00900
BBB	0,00178	0,02579	0,21347	0,49728	0,15021	0,07276	0,01012	0,02859
BB	0,00149	0,01024	0,06298	0,21209	0,34570	0,22432	0,03401	0,10909
B	0,00089	0,00554	0,01936	0,05597	0,16213	0,40709	0,06521	0,28342
CCC	0,00438	0,00357	0,01528	0,04291	0,07651	0,18328	0,09520	0,57905
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

7.year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,51710	0,33221	0,11450	0,02290	0,00780	0,00392	0,00052	0,00104
AA	0,02872	0,53478	0,32280	0,07829	0,01803	0,01177	0,00160	0,00401
A	0,00600	0,09602	0,58562	0,21182	0,05654	0,02833	0,00362	0,01204
BBB	0,00215	0,03030	0,22717	0,45629	0,15387	0,08155	0,01165	0,03699
BB	0,00170	0,01227	0,07370	0,21605	0,30546	0,22426	0,03491	0,13157
B	0,00107	0,00639	0,02360	0,06489	0,16161	0,36213	0,06056	0,31933
CCC	0,00426	0,00440	0,01793	0,04611	0,07815	0,17097	0,07002	0,60832
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

8.year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,47201	0,34691	0,13508	0,02929	0,00943	0,00505	0,00068	0,00154
AA	0,03014	0,49479	0,34057	0,09078	0,02222	0,01410	0,00193	0,00546
A	0,00672	0,10164	0,55379	0,22162	0,06308	0,03318	0,00440	0,01556
BBB	0,00254	0,03462	0,23762	0,42178	0,15536	0,08895	0,01299	0,04612
BB	0,00191	0,01432	0,08358	0,21699	0,27335	0,22083	0,03509	0,15381
B	0,00123	0,00726	0,02794	0,07259	0,15867	0,32416	0,05571	0,35199
CCC	0,00410	0,00520	0,02053	0,04879	0,07827	0,15807	0,05321	0,63199
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

9.year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,43119	0,35698	0,15504	0,03619	0,01126	0,00630	0,00086	0,00217
AA	0,03117	0,45912	0,35443	0,10270	0,02652	0,01659	0,00229	0,00718
A	0,00738	0,10613	0,52589	0,22896	0,06897	0,03794	0,00517	0,01955
BBB	0,00292	0,03869	0,24544	0,39256	0,15532	0,09503	0,01413	0,05587
BB	0,00211	0,01638	0,09258	0,21587	0,24740	0,21519	0,03476	0,17557
B	0,00138	0,00813	0,03230	0,07908	0,15417	0,29177	0,05101	0,38170
CCC	0,00393	0,00597	0,02305	0,05100	0,07728	0,14545	0,04179	0,65167
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

10.year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,39422	0,36318	0,17415	0,04347	0,01330	0,00768	0,00106	0,00295
AA	0,03187	0,42722	0,36502	0,11395	0,03086	0,01921	0,00268	0,00917
A	0,00800	0,10965	0,50132	0,23431	0,07422	0,04254	0,00594	0,02401
BBB	0,00330	0,04251	0,25117	0,36765	0,15422	0,09992	0,01509	0,06611
BB	0,00231	0,01842	0,10068	0,21337	0,22615	0,20817	0,03405	0,19671
B	0,00151	0,00902	0,03660	0,08441	0,14871	0,26389	0,04660	0,40876
CCC	0,00376	0,00670	0,02547	0,05279	0,07554	0,13359	0,03386	0,66843
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

11.year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,36070	0,36619	0,19223	0,05103	0,01551	0,00917	0,00127	0,00388
AA	0,03230	0,39866	0,37289	0,12446	0,03520	0,02195	0,00309	0,01145
A	0,00856	0,11237	0,47961	0,23806	0,07887	0,04693	0,00668	0,02891
BBB	0,00367	0,04605	0,25520	0,34628	0,15240	0,10373	0,01587	0,07674
BB	0,00250	0,02043	0,10791	0,20998	0,20848	0,20034	0,03309	0,21711
B	0,00163	0,00991	0,04079	0,08869	0,14272	0,23972	0,04256	0,43346
CCC	0,00359	0,00739	0,02778	0,05420	0,07329	0,12267	0,02822	0,68299
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

12.year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,33031	0,36656	0,20917	0,05879	0,01790	0,01078	0,00151	0,00498
AA	0,03250	0,37302	0,37849	0,13421	0,03947	0,02476	0,00351	0,01403
A	0,00908	0,11441	0,46031	0,24053	0,08296	0,05108	0,00739	0,03422
BBB	0,00404	0,04931	0,25788	0,32784	0,15010	0,10661	0,01649	0,08766
BB	0,00269	0,02238	0,11430	0,20603	0,19360	0,19212	0,03197	0,23673
B	0,00174	0,01080	0,04483	0,09203	0,13649	0,21863	0,03890	0,45604
CCC	0,00343	0,00804	0,02997	0,05526	0,07073	0,11275	0,02410	0,69584
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

13. year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,30273	0,36478	0,22490	0,06665	0,02043	0,01249	0,00176	0,00626
AA	0,03251	0,34997	0,38221	0,14317	0,04365	0,02762	0,00395	0,01690
A	0,00955	0,11589	0,44310	0,24197	0,08653	0,05496	0,00806	0,03993
BBB	0,00439	0,05230	0,25948	0,31182	0,14750	0,10868	0,01697	0,09881
BB	0,00287	0,02427	0,11992	0,20176	0,18090	0,18379	0,03075	0,25554
B	0,00184	0,01169	0,04867	0,09455	0,13022	0,20011	0,03561	0,47675
CCC	0,00328	0,00866	0,03202	0,05601	0,06800	0,10380	0,02100	0,70734
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

14.year

Current rating	The likelihood of a credit migration to any other rating at the risk horizon (%)							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0,27769	0,36126	0,23938	0,07452	0,02308	0,01431	0,00203	0,00773
AA	0,03237	0,32920	0,38437	0,15136	0,04770	0,03052	0,00441	0,02007
A	0,00998	0,11688	0,42766	0,24258	0,08962	0,05857	0,00869	0,04600
BBB	0,00473	0,05502	0,26020	0,29781	0,14471	0,11004	0,01731	0,11010
BB	0,00305	0,02608	0,12482	0,19734	0,16992	0,17554	0,02949	0,27353
B	0,00193	0,01257	0,05229	0,09634	0,12406	0,18378	0,03266	0,49578
CCC	0,00315	0,00925	0,03394	0,05649	0,06520	0,09575	0,01860	0,71774
D	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	1,00000

Enclosure 3 The spot interest rates

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
AAA	2,810%	3,091%	3,374%	3,657%	3,942%	4,228%	4,516%	4,804%	5,094%	5,386%	5,679%	5,973%	6,269%	6,566%
AA	2,810%	3,095%	3,382%	3,670%	3,959%	4,250%	4,543%	4,837%	5,133%	5,431%	5,730%	6,032%	6,335%	6,640%
A	2,841%	3,130%	3,422%	3,716%	4,014%	4,314%	4,616%	4,921%	5,229%	5,539%	5,852%	6,166%	6,482%	6,800%
BBB	2,903%	3,218%	3,538%	3,862%	4,188%	4,518%	4,850%	5,185%	5,521%	5,860%	6,199%	6,539%	6,879%	7,219%
BB	3,364%	3,789%	4,207%	4,619%	5,025%	5,426%	5,824%	6,217%	6,605%	6,993%	7,375%	7,751%	8,121%	8,485%
B	5,645%	6,108%	6,568%	7,017%	7,455%	7,881%	8,296%	8,700%	9,093%	9,483%	9,860%	10,223%	10,574%	10,914%
CCC	15,561%	15,422%	15,306%	15,171%	15,038%	14,925%	14,843%	14,795%	14,782%	14,815%	14,869%	14,940%	15,026%	15,124%
$\mathbf{r^F}$	2,810%	3,091%	3,372%	3,655%	3,938%	4,222%	4,506%	4,792%	5,078%	5,365%	5,652%	5,941%	6,230%	6,520%
$\mathbf{(1+r^F)^n}$	1,0281	1,0628	1,1046	1,1544	1,2130	1,2816	1,3614	1,4542	1,5617	1,6864	1,8309	1,9988	2,1939	2,4213
R	51,13%													